AMERICAN MATHEMATICAL SOCIETY

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Notices

Edited by J. H. CURTISS

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MEETINGS

CALENDAR OF MEETINGS

NOTE: This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the NOTICES was sent to press. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

Meeting No.	Date	Place	Deadline for Abstracts*
550	November 20-21, 1958	Claremont, California	Oct. 8
551	November 28-29, 1958	Durham, North Carolina	Oct. 8
552	November 28-29, 1958	Evanston, Illinois	Oct. 8
553	January 20-22, 1959 (65th Annual Meeting)	Philadelphia, Pennsylvania	Dec. 5
554	February 28, 1959	New York, New York	Jan. 15
	April, 1959	Monterey, California	
	April 23-25, 1959	New York, New York	
	June 20, 1959	Eugene, Oregon	
	Summer Meeting, 1959	Salt Lake City, Utah	
	November, 1959	Detroit, Michigan	

*The abstracts of papers to be presented at the meetings must be received in the Headquarters Offices of the Society in Providence, R. I., on or before these deadlines. The deadlines also apply to news items.

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FIVE HUNDRED FORTY-NINTH MEETING

Princeton University

Princeton, New Jersey
October 25, 1958

PROGRAM

The five hundred forty-ninth meeting of the American Mathematical Society will be held at Princeton University, Princeton, New Jersey, on Saturday, October 25, 1958. All times given below are Eastern Daylight Saving Time (Princeton returning to Eastern Standard Time on Sunday, October 26).

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, Professor Arnold Shapiro of Brandeis University will give an hour address entitled "Imbedding a complex in a cell" at 2:00 P.M. in Room 301 of the Palmer Physical Laboratory.

There will be sessions for contributed papers at 10:30 A.M. and 3:15 P.M. in Rooms 301 and 303 of the Palmer Physical Laboratory and Room 100 of Fine Hall. Abstracts of the papers to be presented in these sessions appear on pages 596-609 of these NOTICES. There are cross references to the abstracts in the program. Thus, for example, the title of paper (1) in the program is followed by (549-11), indicating that the abstract can be found under the designation 549-11 among the published abstracts.

The registration desk will be located on the first floor of Fine Hall. Information about meals may be obtained there.

The Fine Hall lounge and Rooms 113 and 114 of Fine Hall will be available for informal conversations.

The meeting will be held in Fine Hall and the Palmer Physical Laboratory, which are adjoining buildings on the Princeton University campus. Princeton can be reached by car using U. S. 1 or U. S. 206. One turns off U. S. 1 at the Penns Neck traffic circle on Washington Road. Palmer Laboratory is on the west side of Washington Road, near the corner of Prospect Street. A campus policeman will be on duty to direct people for parking.

Railroad service to Princeton is via the Pennsylvania Railroad, stopping at Princeton Junction with a connecting shuttle train to Princeton. The Pennsylvania Railroad has very kindly arranged for a special stop at Princeton Junction on October 25 of train no. 109 (leaving New York City at 8:30 A.M.) in addition to the regularly scheduled stops of other trains.

Bus service between Princeton and the Port of New York

Authority, 41st Street and 8th Avenue, New York City, is available via the Suburban Transit Corporation.

Mail and telegrams for those attending the meeting should be addressed to: Department of Mathematics, Fine Hall, Box 708, Princeton, New Jersey.

PROGRAM OF THE SESSIONS

(Time limit for each contributed paper, 10 minutes)

SATURDAY, 10:30 A.M.

Session on Analysis, Palmer Physical Laboratory, Room 301

(1) Closed extensions of unbounded linear transformations
Dr. R. D. McWilliams, Princeton University (549-11)

(2) On the spectral theory of singular integral operators
Dr. Walter Koppelman, Yale University (549-8)

(3) On the "product" of semi-groups of operators
Professor H. F. Trotter, Queen's University (549-23)

(4) Integration and nonlinear transformations in Hilbert space
Dr. Leonard Gross, Yale University (549-20)

(5) Representations of *-algebras
Dr. R. T. Prosser, Massachusetts Institute of Technology (549-19)

(6) Multiplication and division of distributions
Professor I. S. Gál, Cornell University and Yale University (549-7)

Session on Analysis, Applied Mathematics, Statistics and Probability, Palmer Physical Laboratory, Room 303

(7) About the concept of meromorphic maps
Dr. W. F. Stoll, Institute for Advanced Study and University of Tübingen (549-4)

(8) Boundary images of meromorphic functions
Professor P. T. Church, Syracuse University (549-18)

(9) On the growth of meromorphic functions
Professor Albert Edrei, Syracuse University, and Professor W. H. J. Fuchs, Cornell University (549-26)

(10) Stability criteria for the numerical solution of partial differential equations by boundary contraction
Dr. H. W. Milnes and Dr. R. B. Potts, General Motors Corporation, Detroit, Michigan (549-6)

(11) On the Liénard potentials
Professor Domina E. Spencer, University of Connecticut (549-22)

(12) A probability limit theorem requiring no moments
Professor J. G. Kemeny, Dartmouth College (549-1)

Session on Topology, Logic and Foundations, Fine Hall, Room 100

(13) Harmonic integrals on foliated manifolds
Dr. B. L. Reinhart, University of Michigan (549-2)

(14) On clusters in proximity spaces
Professor Solomon Leader, Rutgers, The State University (549-10)

(15) Product spaces in n-manifolds
Professor F. B. Jones, University of North Carolina, and Professor G. S. Young, University of Michigan (549-16)

(16) Runs
Professor Hewitt Kenyon, University of Rochester, and Professor A. P. Morse, University of California, Berkeley (549-9)

(17) Filtration systems. I
Professor S. B. Kochen, University of Montreal (549-24)

(18) Uniqueness ordinals and extended R-systems
Professor Hilary Putnam, Princeton University, and Professor Martin Davis, Rensselaer Polytechnic Institute, Hartford Graduate Center (549-30)

SATURDAY, 2:00 P.M.

Invited Address, Palmer Physical Laboratory, Room 301

Imbedding a complex in a cell (One hour)
Professor Arnold Shapiro, Brandeis University

SATURDAY, 3:15 P.M.

Session on Analysis, Palmer Physical Laboratory, Room 301

(19) On Diophantine approximation and trigonometric interpolating polynomials
Professor R. P. Gosselin, University of Connecticut (549-28)

(20) Some polynomial extremal problems and mechanical quadratures
Professor I. J. Schoenberg, University of Pennsylvania (549-32)

(21) Extremal measures, linear functionals, and Tchebycheff approximation
Professor H. S. Shapiro, New York University (549-3)

(22) The kernel-function and conformal invariants
Professor Bernard Epstein, University of Pennsylvania (549-12)

(23) A singular integral whose kernel involves a Bessel function. II
Professor Lee Lorch, Philander Smith College and Wesleyan University, and Professor Peter Szego, University of Santa Clara (549-15)

(24) Quasi additive set functions and the concept of integral
Dr. Lamberto Cesari, Purdue University (549-33)

(25) Solution of a problem proposed by I. M. Sheffer
Mr. L. B. Robinson, Baltimore, Maryland (549-13)

Session on Algebra and Theory of Numbers, Palmer Physical Laboratory, Room 303

(26) Quadratic lattices over local fields
Professor George Whaples, Indiana University (549-17)

(27) Multiplicative ideal theory of a non-commutative ring
Professor G. D. Findlay and Professor Joachim Lambek, McGill University (549-27)

(28) Rings of quotients of Boolean rings
Mr. B. Brainerd, University of Western Ontario, and Professor Joachim Lambek, McGill University (549-14)

(29) On Lie algebras of rank one
Dr. Richard Block, Yale University (549-25)

(30) Geometric proofs of two theorems in group theory. Preliminary report
Dr. Leon Greenberg, Brown University (549-5)

(31) Groups with L-polynomials
Dr. Elvira S. Rapaport, Stockbridge, Massachusetts (549-21)

(32) Generalizations of Brauer theorems on stochastic matrices
Dr. Emilie V. Haynsworth, National Bureau of Standards, Washington, D. C. (549-29)

(33) On the determination of numbers by their sums of a fixed order
Dr. J. L. Selfridge, IBM Research Center, Yorktown Heights, New York and University of California, Los Angeles, and Professor E. G. Straus, University of California, Los Angeles (549-31)

SUPPLEMENTARY PROGRAM
(To be presented by title)

(34) On the solution of simultaneous first order implicit differential equations
Dr. Smbat Abian and Professor A. B. Brown, Queens College

(35) Multi-copy generalized networks
Professor Abraham Charnes, Northwestern University, and Professor C. E. Lemke, Rensselaer Polytechnic Institute

(36) A trigonometric sum
Professor Eckford Cohen, University of Tennessee

(37) Self adjoint boundary conditions for the Laplacian. I. Bounded regions in the pane
Mr. R. S. Freeman, University of California, Livermore

(38) Self adjoint boundary conditions for the Laplacian. II. Unbounded regions
Mr. R. S. Freeman, University of California, Livermore

(39) Some properties of the Dirac delta
Professor I. S. Gál, Cornell University and Yale University

(40) A discreteness theorem for subgroups of the Lorentz group. Preliminary report
Dr. Leon Greenberg, Brown University

(41) Hyperbolic space forms. Preliminary report
Dr. Leon Greenberg, Brown University

(42) Filtration systems. II
Professor S. B. Kochen, University of Montreal

(43) On harmonic functions of three variables
Professor E. O. A. Kreyszig, Ohio State University

(44) The radius of univalence of Bessel functions. I
Professor Erwin Kreyszig, Ohio State University, and Professor John Todd, California Institute of Technology

(45) 2 x 2 matrices of trace 0
Mr. D. B. Lissner, Cornell University

(46) Linear differential equations and functional analysis. V
Professor J. L. Massera and Professor J. J. Schäffer, Universidad de Montevideo

(47) Linear differential equations and functional analysis. VI
Professor J. L. Massera and Professor J. J. Schäffer, Universidad de Montevideo

(48) On the existence of a scattering operator
Dr. R. T. Prosser, Massachusetts Institute of Technology

(49) Function spaces with translations
Professor J. J. Schäffer, Universidad de Montevideo
(Introduced by Professor J. L. Massera)

(50) General boundary value problems for elliptic partial differential equations. I
Dr. Martin Schechter, New York University

(51) General boundary value problems for elliptic partial differential equations. II
Dr. Martin Schechter, New York University

(52) On integration of 1-forms
Professor Maurice Sion, University of California, Berkeley

(53) Metrisability of unions of spaces
Dr. A. H. Stone, University of Manchester

(54) Note on least-square approximation to an analytic function by polynomials, as measured by a surface integral
Professor J. L. Walsh, Harvard University

(55) Compact transformation groups of n-sphere with an (n - 1)-dimensional orbit
Professor Hsien-Chung Wang, Northwestern University

(56) Inversion of Toeplitz matrices. II
Professor Harold Widom, Cornell University

(57) Strong converse of the coding theorem for discrete finite-memory channels
Professor Jacob Wolfowitz, Cornell University

(58) Concerning dense metric subspaces of certain nonmetric spaces
Mr. J. N. Younglove, University of Texas

R. D. Schafer
Associate Secretary

Princeton, New Jersey
September 11, 1958

PRELIMINARY ANNOUNCEMENTS OF MEETINGS

FIVE HUNDRED AND FIFTIETH MEETING

Pomona College
Claremont, California
November 21-22, 1958

The five hundred and fiftieth meeting of the American Mathematical Society will be held on Friday and Saturday, November 21-22, 1958, at Pomona College, Claremont, California.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, A Symposium on Asymptotic Expansions will be held on Friday. The Program Committee for the Symposium consists of Professors Arthur Erdélyi, chairman, J. G. van der Corput, R. E. Langer, and T. E. Hull.

By invitation of the same Committee, there will be an address on Saturday morning by Professor Leon Henkin of the University of California, Berkeley, on "Some recent work on real closed fields". Sessions for contributed papers will follow on Saturday afternoon.

Members of the Society and their friends are invited to teas on Friday and Saturday afternoons. A recital of the Claremont Trio is planned on Friday evening.

Claremont can be reached by bus from Sixth and Main Streets, or by automobile via the San Bernardino Freeway (U.S. 99) from Los Angeles.

Midday dinner on Saturday will be available at the Gibson Dining Hall on campus. There are numerous restaurants in and near Claremont.

Overnight accommodations can be reserved at the following conveniently located hotels: Claremont Inn, third and College Avenue, Claremont; Holiday Inn Hotel, San Bernardino Freeway at Central Avenue, Claremont. Reservations for rooms should be made directly with the chosen hotel at least two weeks in advance of the meeting.

R. S. Pierce
Acting Associate Secretary

Seattle, Washington
September 1, 1958

FIVE HUNDRED FIFTY-FIRST MEETING

Duke University
Durham, North Carolina
November 28-29, 1958

The five hundred fifty-first meeting of the American Mathematical Society will be held at Duke University, Durham, North Carolina, on November 28-29, 1958, in the Physics Building on West Campus.

By invitation of the Committee to Select Hour Speakers for Southeastern Sectional Meetings, Professor E. E. Floyd of the University of Virginia will address the Society on "Compact groups of transformations" at 2:00 P.M. Friday in Room 114, Physics.

Sessions for contributed papers will be held at 3:30 to 5:00 P.M. on Friday and 9:00 A.M. to noon on Saturday.

Registration headquarters will be in the Physics Building, and will be open from 10:00 A.M. Friday through noon Saturday. Tea will be served at 4:00 P.M. Friday. There will be a banquet Friday at 6:30 P.M., the charge being $2.00 per person. A cafeteria on campus will be available for breakfast for the exclusive use of members of the Society and their guests on Saturday morning, from 8:00 to 9:00 A.M. Coffee will also be served Saturday morning in the Physics Building. Banquet reservations should be made by card or letter to Professor R. M. McLeod, Department of Mathematics, Duke University, Durham, North Carolina, and should reach him by November 14. Reservations for rooms should be made directly with the chosen hotel or motel. (Downtown Durham is approximately 3 miles from the West Campus of Duke University.) Bus service is available between downtown Durham and West Campus. Ample parking space is available near the Physics Building. Minimum rates are listed below for conveniently located hotels and motels:

	Single	Double
Washington Duke Hotel Corcoran and Chapel Hill Sts., Durham	$4.50	$ 8.25
Biltmore Hotel (colored) 332 East Pettigrew, Durham	3.00	4.00
Eden Rock Motel, 3 miles South of Durham on Highways 15A and 501 (3 1/2 miles from the Physics Building)	---	10.00
Homestead Motel, 2 miles south of Durham on Highways 15A and 501 (3 miles from Physics Building)	7.00	8.00

El Rancho Motel, Elf Street, Durham, near Veterans' Hospital (3/4 miles from Physics Building) $7.00 $9.00

Mail and telegrams for those attending the meeting may be sent in care of the Mathematics Department, Physics Building, Duke University, Durham, North Carolina.

G. B. Huff
Associate Secretary

Athens, Georgia
September 11, 1958

FIVE HUNDRED FIFTY-SECOND MEETING

Northwestern University
Evanston, Illinois
November 28-29, 1958

The five hundred fifty-second meeting of the American Mathematical Society will be held at Northwestern University, Evanston, Illinois, on Friday and Saturday, November 28-29, 1958. All sessions will be held in the Technological Institute.

The Committee to Select Hour Speakers for Western Sectional Meetings has invited Professor Roger Lyndon of the University of Michigan to address the Society. Professor Lyndon will speak on "Properties preserved under algebraic operations", at 2:00 P.M. in Lecture Room 2 of the Northwestern Technological Institute Building.

Sessions for the presentation of contributed papers will be held at 10:30 A.M. on Friday and Saturday, and at 3:15 P.M. on Friday. If necessary, there will be special sessions for the presentation of contributed papers which failed to meet the deadline. If such sessions are held, they will be publicized by a special program available at the registration desk.

The registration desk will be located in the front lobby of the Northwestern Technological Institute Building located on Sheridan Road at Noyes Street. Those who attend the meetings are requested to register at any time from 9:30 A.M. to 5:00 P.M. on Friday, and from 9:30 A.M. to noon Saturday.

The Orrington Hotel, 1710 Orrington Avenue, Evanston, has agreed to hold a block of rooms for Society members until November 13, 1958. Members should write directly to the Orrington. Single rooms range from $7.00 to $12.00 and double rooms from $10.00 to $14.00. The North Shore Hotel in Evanston has comparable prices.

Complete details will be found in the final program in the next issue of the NOTICES.

J. W. T. Youngs
Associate Secretary

Bloomington, Indiana
September 11, 1958

NEWS ITEMS AND ANNOUNCEMENTS

FELLOWSHIP AND RESEARCH OPPORTUNITIES IN MATHEMATICS DURING THE YEAR 1959-1960. The Division of Mathematics of the National Academy of Sciences-National Research Council has issued the following annual announcement concerning fellowship and research opportunities during the year 1959-1960. The announcement is intended to provide only a partial list of the various opportunities which are available.

1. National Science Foundation. The National Science Foundation sponsors various fellowship programs in the sciences, including mathematics.

Predoctoral fellowships are awarded annually at the First Year, Intermediate, and Terminal Year levels of graduate study. Applications for 1959-1960 will be available in October 1958 from the National Academy of Sciences - National Research Council until the closing date in early January 1959; Award date--March 16, 1959.

Science Faculty fellowships for college science teachers (including mathematics) who plan to continue teaching and wish to increase their competence as teachers are at the present time offered semi-annually. Eligibility requirements include a baccalaureate degree and three (3) years of full-time experience in teaching natural science subjects at the collegiate level. Awarded annually. The program will be open from May to October. Awards will be announced in early December. Address all inquiries for information and applications to National Science Foundation, Division of Scientific Personnel and Education, Washington 25, D. C.

Postdoctoral fellowships (in making inquiry about postdoctoral awards specify program).

(1) Regular postdoctoral fellowships--primarily for recipients of the doctoral degree; awarded semi-annually. Program for 1959-1960 concurrent with predoctoral program (see above) except that program closes in December. Information and applications will be available from the NAS-NRC. The program will also be open from July to early September 1959. Awards are announced in March and October.

(2) Senior postdoctoral fellowships--are open to persons who have held a doctoral degree in one of the basic fields of science for a minimum of five (5) years at time of application, or who have had equivalent training and experience. Awarded annually. Applications are available from the National Science Foundation, Division of Scientific Personnel and Education, Washington 25, D. C. The program will be open from May to October. Awards will be announced in early December.

Research Grants. The National Science Foundation also supports basic research in the mathematical sciences by means of grants. While proposals for such support are accepted at any time, individuals desiring support to begin the summer or at the beginning of a fall semester should preferably submit their proposals in the mathematical sciences by November 1; persons desiring support to begin in the spring semester should preferably submit their proposals by May 1. Instructions for the preparation of proposals, contained in a booklet entitled Grants for Scientific Research, may be obtained upon request from the Program Director for Mathematical Sciences, National Science Foundation, Washington 25, D. C.

2. Office of Naval Research. The Office of Naval Research, through contracts with universities and other organizations, supports basic research in broadly selected fields of mathematics. Proposals should be directed to the Mathematics Branch, Office of Naval Research, Washington 25, D. C. In addition, postdoctoral research associateships in pure mathematics are being established under contracts with the ONR at selected universities. For details and application forms write to the above address.

3. Air Force Office of Scientific Research. The Air Force Office of Scientific Research supports research in mathematics directly through contracts with colleges, universities, foundations and industrial laboratories. Such organizations are encouraged to submit proposals for research in mathematical fields in which they specialize. Proposals should be mailed to the Commander, Air Force Office of Scientific Research, Attn: Mathematics Division, Washington 25, D. C.

4. Office of Ordnance Research, U. S. Army. Among the functions of the Office of Ordnance Research is the support of basic research in mathematics. Proposals for projects are ordinarily made by individual scientists or groups of scientists in a form which leads to a contract between the Office of Ordnance Research and a university or research laboratory. For further information write to Commanding Officer, Office of Ordnance Research, Box CM, Duke Station, Durham, North Carolina.

5. Fulbright Awards - Public Law 584 (79th Congress). Approximately 400 awards are offered annually for university lecturing and postdoctoral research in all academic fields in Argentina, Australia, Brazil, Burma, Chile, Colombia, Ecuador, India, New Zealand, Pakistan, Paraguay, Peru, the Philippines and Thailand (competition for the preceding countries closes April 15, 1959); Austria, Belgium-Luxembourg, Republic of China, Denmark, Finland, France, Germany, Greece, Iceland, Iran, Ireland, Israel, Italy, Japan, the Netherlands, Norway, Turkey, and the United Kingdom including colonial dependencies (competition for the latter countries closes October 1, 1959).

In both cases awards are for the academic year 1960-61 (the 1959-60 competition for Europe closes October 1, 1958), but in the former group of countries the academic year begins in the spring or summer instead of the autumn. Awards are payable in foreign currency and usually include travel for the grantee, but not for members of his family, and a maintenance allowance, which may be adjusted in relation to the number of accompanying dependents up to four. Requests for information should be addressed to the Committee on International Exchange of Persons, Conference Board of Associated Research Councils, 2101 Constitution Avenue, Washington 25, D. C.

6. National Bureau of Standards. Naval Research Laboratory. Air Research and Development Command. Postdoctoral resident research associateships are available in a variety of sciences including mathematics and are tenable at the Washington, D. C. and Boulder, Colorado laboratories of the National Bureau of Standards; at the Naval Research Laboratory in Washington, D. C.; and at selected development and research centers of the Air Research and Development Command. Necessary facilities and equipment incident to the research of the associate will be provided. For further information write to Fellowship Office, National Academy of Sciences - National Research Council, 2101 Constitution Avenue, Washington 25, D. C. Applications for the 1959-60 program must be filed on or before January 19, 1959.

7. Atomic Energy Commission. The Division of Research of the Atomic Energy Commission through contracts with universities and other organizations supports research in the fields of numerical analysis, digital computer design, programming research, and related topics. Proposals should be submitted to the Division of Research, Atomic Energy Commission, Washington 25, D. C.

Brookhaven National Laboratory. Brookhaven National Laboratory, operated by Associated Universities, Inc. under contract with the Atomic Energy Commission offers postdoctoral research appointments in mathematics. Appointments are for one year, and may be renewed for one additional year. U. S. citizenship is not required, although Atomic Energy Commission approval is a prerequisite. The appointee may work in numerical analysis, digital computing, mathematical physics, differential equations, probability and statistics, and various specialized branches including reactor theory, hydrodynamics, and orbit theory. Computational facilities are available. Letters from candidates should give details of personal history, scientific background, and qualifications; two letters of recommendation, one from the applicant's research professor, are required. Applications should be directed to M. E. Rose, Head, Applied Mathematics Division, Brookhaven National Laboratory, Upton, Long Island, New York.

GRANTS-IN-AID OF THE INSTITUTE FOR ADVANCED STUD

The School of Mathematics of the Institute for Advanced Study will allocate a small number of grants-in-aid to gifted young mathematicians and theoretical physicists to enable them to study and to do research work at Princeton during the academic year 1959-60. Candidates must have given evidence of ability in research comparable at least with that expected for the degree of Doctor of Philosophy. Blanks for application may be obtained from the School of Mathematics, Institute for Advanced Study, Princeton, New Jersey, and are returnable by January 1, 1959.

NATIONAL SCIENCE FOUNDATION SENIOR POSTDOCTORAL AND SCIENCE FACULTY FELLOWSHIPS. The deadline for applications for these fellowships is October 15, 1958. Inasmuch as this is the first year during which these awards will be made annually rather than biannually, applications for awards for 1959 must be submitted by the October closing date. Announcement of awards will be made on December 5, 1958 by the Foundation.

Awards will be made in the mathematical, physical, and engineering sciences; medical and biological sciences including anthropology and psychology (excluding clinical); and in selected social science fields. Included as well are interdisciplinary fields which overlap two or more scientific disciplines.

Candidates for Senior Postdoctoral Fellowships must be United States citizens who have held the doctoral degree for a minimum of five years or have equivalent education and experience. Under arrangements made by the National Academy of Sciences, candidates' qualifications will be evaluated by carefully chosen panels of scientists. Final selection of approximately 100 Fellows will be made by the National Science Foundation.

The Science Faculty Fellowships are directed toward college teachers of science who wish to improve their competence as teachers. These fellowships are open to application by any United States citizen who holds a baccalaureate degree or its equivalent, has demonstrated ability and special aptitude for science teaching and advanced training, and has taught at the collegiate level as a full-time faculty member for a minimum of three years and intends to continue teaching. Under arrangements made by the Association of American Colleges, applicants' qualifications will be evaluated by persons especially competent to make judgments as to the demonstrated and potential ability of the applicant as a teacher of science. Final selection of approximately 300 Fellows will be made by the National Science Foundation.

Application materials may be obtained from the Division of Scientific Personnel and Education, National Science Foundation, Washington 25, D. C. Completed materials must be received not later than October 15, 1958.

GRADUATE FELLOWSHIPS OFFERED BY THE AMERICAN ASSOCIATION OF UNIVERSITY WOMEN FOR 1959-1960. Quite a large number of graduate fellowships are offered every year by the AAUW. They are open to women who are American citizens, unless otherwise stated. They are awarded in general to candidates who have already received the doctorate or who will have fulfilled all the requirements for the degree except the dissertation by the time the fellowship year begins (July 1). The greatest importance is attached to the project on which the candidate wishes to work - its significance and the evidence of the candidate's ability to pursue it. The candidate must show promise of distinction in her field of study.

These fellowships are described at some length in a pamphlet which is published by the AAUW. This pamphlet and additional information may be obtained by writing to the Committee on Fellowship Awards, American Association of University Women, 1634 Eye Street, N. W., Washington 6, D. C. Applications and recommendations for these fellowships must reach the Secretary of the Committee on Fellowship Awards at the above address not later than December 1, 1958. The pamphlet referred to above contains directions for applying for the fellowships and general conditions of acceptance.

OPPORTUNITY FELLOWSHIPS FOR 1959-1960 OF THE JOHN HAY WHITNEY FOUNDATION. The purpose of these fellowships is to give opportunities for special experience or advanced study to outstanding individuals who otherwise might not be able to reach their fullest development or make their fullest contribution. The competition for Opportunity Fellowships is open to citizens of the United States (including residents of territories) who have given evidence of special ability and who have not had full opportunity to develop their talents because of arbitrary barriers, such as racial or cultural background or region of residence. Candidates are expected to be mature enough to have given positive evidence of superior promise, yet young enough to have their careers before them; in general to be between the ages of 22 and 35 and to have completed their undergraduate college education. The fellowships are open not only for academic study (graduate) but for other types of training or experience which may be most useful in developing varied talents and varied forms of leadership. Awards are for a full year of serious work, not for incidental or temporary projects. In special cases grants may be renewed for a second year. Awards are expected normally to range from $1,000 to $3,000 depending on the nature of the proposed project and the financial need of the candidate. It is hoped that in many cases funds from other sources may supplement these awards (for example, fellowships for study in foreign countries, additional scholarship aid from universities, payments for certain types of apprentice work, etc.). The awards are made annually on the basis of for-

mal written applications by the candidates on forms provided by the John Hay Whitney Foundation. Complete applications must be filed not later than November 30. The results of the competition will be announced in the latter part of April. Inquiries should be addressed to Opportunity Fellowships, John Hay Whitney Foundation, 630 Fifth Avenue, New York 20, New York.

AN INTERDISCIPLINARY CONFERENCE ON SELF-ORGANIZING SYSTEMS will be held on May 5 and 6, 1959 at the Museum of Science and Industry, Chicago, Illinois. The sponsors are the Information Systems Branch of the Office of Naval Research and Armour Research Foundation of Illinois Institute of Technology. The topics to be discussed at the conference include the general theory of the learning and cognitive processes, as well as its applications to the design of automata having properties frequently referred to as adaptive. Such recent developments as the "Perceptron" undoubtedly will find their place on the program. Further information may be obtained from Scott H. Cameron, Electrical Engineering, Research Department of Armour Research Foundation, 10 West 35th Street, Chicago 16, Illinois.

THE FIFTH ANNUAL COMPUTER APPLICATIONS SYMPOSIUM will be held in Chicago at the Morrison Hotel on October 29 and 30. It will be sponsored by the Armour Research Foundation of Illinois Institute of Technology. It will stress future areas of computer applications. The discussions will cover such subjects as automatic programming techniques, learning procedure simulation, and applications of new data processors and computers. The program for the first day will emphasize business and management applications. Mr. Fred Gruenberger of RAND Corporation will be chairman of this session. The program for the second day will be devoted to engineering and scientific applications. Dr. C. B. Tompkins, professor of mathematics at the University of California at Los Angeles, will be the chairman of this session. Topics scheduled for consideration at the business and management session include new concepts in large scale data processing, sharing a data processing facility by several organizations, and inventory and logistics problems. At the engineering and scientific applications session, the future of automatic programming, programming of learning techniques, and literature searching are among the topics to be discussed. Fourteen speakers will participate in the two-day meeting. Inquiries concerning the conference should be sent to Dr. Frederick Bock, Electrical Engineering Research Department, Armour Research Foundation of Illinois Institute of Technology, 10 West 35th Street, Chicago 16, Illinois.

THE 1958 SUMMER INSTITUTE. This Summer Institute was as usual supported by the National Science Foundation. The topic this summer was "Surface Area and Related Topics". The Institute was held on the campus of Bowdoin College, Brunswick, Maine, during the four-week period beginning Monday, June 16, 1958. The following mathematicians were in attendance: Miriam C. Ayer (University of Missouri), L. Cesari (Purdue University), D. E. Christie (Bowdoin College), W. G. Collar (University of Wisconsin), J. M. Danskin (Institute for Advanced Study), M. R. Demers (University of Nevada), H. Federer (Brown University), W. H. Fleming (Purdue University), G. Freilich (City College, New York), R. E. Fullerton (University of Maryland), R. P. Goblirsch (University of Virginia), C. Goffman (Purdue University), R. E. Hughs (Purdue University), A. T. Lauria (Purdue University), E. J. Mickle (Ohio State University), W. C. Nemitz (Ohio State University), T. Nishiura (Purdue University), C. J. Neugebauer (Purdue University), T. Rado (Ohio State University), A. W. Ransom (Purdue University), R. Rishel (University of Wisconsin), J. C. Sanwal (Indiana University), E. Silverman (Purdue University), D. Sokolowski (Antioch College), L. H. Turner (Purdue University), H. Whitney (Institute for Advanced Study), R. F. Williams (Purdue University), L. C. Young (University of Wisconsin), J. W. T. Youngs (Indiana University).

HEARINGS BEFORE THE RESEARCH AND DEVELOPMENT SUBCOMMITTEE OF THE JOINT COMMITTEE ON ATOMIC ENERGY. Last February the Research and Development Subcommittee of the joint Committee on Atomic Energy of the Congress of the United States held extensive public hearings on physical research as it relates to the field of atomic energy. Some 50 leading research scientists appeared before the Subcommittee to describe research activities in the major areas of physics, chemistry and metallurgy.

These hearings have now been printed by the Government Printing Office. They run to 785 pages. The testimony includes interesting statements concerning the impact of mathematics and automatic computers on the physical sciences by Dr. N. C. Metropolis and Dr. R. D. Richtmyer, as well as other material of interest to mathematicians. A limited number of copies of this publication has been forwarded to the Headquarters Offices of the American Mathematical Society, and are available while the supply lasts for distribution to members upon request. Postage will be paid by the Society.

RUSSIAN SCIENTIFIC JOURNALS AND JOURNAL ARTICLES AVAILABLE IN ENGLISH. The National Science Foundation has issued a compilation of the Soviet scientific and technical journals now available in English translation in the United States. It was

indicated that there are now in print 53 English editions of Russian journals, 4 extensive series of translated Russian abstracts of scientific papers, and 4 series of partial translations of important Russian journals. The number of translations of complete journals supported by the National Science Foundation is now 31.

Some of the items which might be of particular interest to mathematicians are as follows:

Title	Available from	Subscription Price Per Year
Journal of Technical Physics (Zhurnal Teknicheskoi Fiziki), Monthly	American Institute of Physics, 335 East 45th Street, New York 17, New York	$75.00 35.00 **
Acoustics Journal (Akousticheskii Zhurnal), Quarterly	"	$12.00
Proceedings of the USSR Academy of Sciences (Doklady Akademii Nauk SSSR), Bimonthly Physics Sections	"	$35.00 15.00 *
Journal of Experimental and Theoretical Physics (Zhurnal eksperimentalnoi i Teoreticheskoi), Monthly [Translation Journal known as Soviet Physics - JETP]	"	$60.00 ** 75.00 *** 35.00 *
Astronomical Journal (Astronomicheski Zhurnal), Bimonthly	"	$25.00 10.00 *
Crystallography (Kristallografiia), Bimonthly	" "	$25.00 10.00 *
American Mathematical Society Translations (Translations of Selected Russian Mathematical Articles)	A-M-S 190 Hope Street Providence 6, Rhode Island	
Series 2, Volume 1	"	$ 3.60
Series 2, Volume 2	"	3.70
Series 2, Volume 3	"	3.30
Series 2, Volume 4	"	3.80
Series 2, Volume 5	"	4.30

Series 2, Volume 6	A-M-S	$ 5.20
Series 2, Volume 7	"	5.50
Series 2, Volume 8	"	6.00
Series 2, Volume 9	"	5.00
Journal of Applied Mathematics and Mechanics (Prikladnaya Matematika i Mekhanika), Bimonthly	Pergamon Institute 122 East 55th Street New York 22, New York or Pergamon Institute 4 and 5 Fitzroy Square London, W. 1, ENGLAND	$35.00
Theory of Probability and its Applications (Teoriya Veroyatnostei i Primenenie), Quarterly	Society for Industrial and Applied Mathematics Box 7541 Philadelphia 1, Pennsylvani	$15.00 6.00 *
Atomic Energy (Atomnaia Energiia), Monthly	Consultants Bureau, Inc. 227 West 17th Street New York 11, New York	$75.00
Applied Physics Section (Otdel Prikladnoi Fiziki),	"	$200.00
Proceedings of the Academy of Sciences of the USSR (Doklady Akademii Nauk SSR) Chemistry Section (Otdel Khimii), Bimonthly	"	$95.00
Atomic Energy (Selected Papers) Atomnaia Energiia), Incorporated in British Jrnl. of Nuclear Energy, 3 volumes	Pergamon Press, Ltd. 122 East 55th Street New York 22, New York or Pergamon Press, Ltd. 4 and 5 Fitzroy Square London, W. 1, ENGLAND	$ 9.80/vol. 20.00/vol.****

* Price per year to libraries of educational institutions.
** Renewal price.
*** New subscription price.
**** Price per year to libraries.

PROFESSOR RICHARD COURANT RETIRES. Professor Richard Courant retired on August 31 as Scientific Director of the Institute of Mathematical Sciences of New York University and Head of the University's Mathematics Department. He will become Professor Emeritus of Mathematics and Science Advisor to the University. On July 18, 1958, he was presented by the Navy Department with the Navy Distinguished Public Service Award. The award is the highest the Navy confers on civilians. Earlier this year Professor Courant received three honorary Doctor of Science degrees: from New York University, from Case Institute of Technology, and from the Technische Hochschule in Aachen, Germany. New York University previously honored Professor Courant last January with the establishment at the University of the Richard Courant Lectureship in Mathematical Sciences.

THE ANNUAL LIST OF CHAIRMEN OF DEPARTMENTS has been compiled. Copies may be obtained free of charge by writing to the Headquarters Offices, 190 Hope Street, Providence 6, Rhode Island.

ANNUAL SALARY SURVEY. In May, 1957, the Society's Committee on the Economic Status of Teachers made a salary survey by means of a questionnaire mailed to the 61 institutions covered in the 1955-56 Survey of Training and Research Potential in the Mathematical Sciences (the so-called Albert Survey). The Council has now directed the Executive Director to make a survey of mathematicians' salaries on an annual basis and publish the results in the NOTICES. It is felt that regular publication of the data will be useful in efforts to obtain satisfactory salaries for mathematicians.

The present survey consists of two parts. In Part I is presented information from new questionnaires mailed in May 1958, to all of the 61 institutions which were covered by the Albert survey, and also from other similar questionnaires mailed to the entire remainder of the Institutional Membership list of the American Mathematical Society. The list of Institutional Members numbers 146, of which 85 were not included in the Albert survey. There were 47 usable replies from the Albert survey institutions and 52 from the non-Albert survey institutions.

In Part II, information is presented which was obtained from an analysis of the salary data currently available in the files of the Mathematical Sciences Section of the National Register of Scientific and Technical Personnel (NRSTP). The Register is a responsibility of the National Science Foundation and the Mathematical Sciences Section is operated for the Foundation by the Society.

The salary data given for college and university teachers in

Part I refer only to the academic year (9-10 months). It was indicated in the questionnaire used for Part I that by salary is meant the payment by the school to the individual for full-time work. Grants and contracts were to be included, but sabbatical payments and other part-time salaries were to be excluded. On the other hand, in the instructions which accompany the NRSTP questionnaires, the definition of salary is perhaps a little less definite as regards university personnel. The questions dealing with salary first ask the respondent to indicate his "annual income from principal employment" as described earlier in the questionnaire, and then to indicate "other professional income in addition to above, from secondary employment(s), summer teaching, royalties, patents, etc.". The data given below in Part II are based only on the answers to the first question, but they still may not be strictly comparable with the data in Part I. Another factor affecting comparability is that in Part II employees of non-profit institutions have been lumped in together with the strictly academic people because it is hard to distinguish from looking at the returns between employees of university research projects or institutes and regular members of academic departments.

All salary figures given below are given in hundreds of dollars and are rounded to the nearest hundred with downward preference for lower extremes and upward preference for upper extremes.

In Part I the raw data would have permitted the more detailed and complex type of summaries given last year in which minima of minima, medians of minima, etc., etc., were tabulated. The presentation here is considerably simplified. The Editor would appreciate comments as to whether the earlier more detailed presentation is preferable.

PART I

DIRECT SURVEY OF ACADEMIC INSTITUTIONS

MAJOR STATE UNIVERSITIES AND COLLEGES

Number of questionnaires mailed: 15
Number of usable returns: 11

Total number on the staffs working full time on the campus

RANK	1957-58 (Actual)	1958-59 (Employed for the academic year as of now)
Instructor (only those holding Ph.D.)	27	15-1/2
Assistant Professor	94-1/3	95
Associate Professor	58-1/2	66-1/3
Professor	92-5/6	97-1/3
TOTAL	272-2/3	274-1/6

Salary Survey

RANK	1957-58 Min.	1957-58 Max.	1958-59 Min.	1958-59 Max.
Instructor (only those holding Ph.D.)	45- 52	50- 60	48- 54	52- 60
Assistant Professor	50- 62	60- 73	52- 62	60- 73
Associate Professor	58- 79	68- 92	58- 79	73- 92
Professor	60-103	94-161	60-103	105-161

MAJOR NON-STATE UNIVERSITIES AND COLLEGES

Number of questionnaires mailed: 8
Number of usable returns: 5

Total number on the staffs working full time on the campus

RANK	1957-58 (Actual)	1958-59 (Employed for the academic year as of now)
Instructor (only those holding Ph.D.)	23	16
Assistant Professor	17	16
Associate Professor	16-1/2	21
Professor	36	37
TOTAL	92-1/2	90

Salary Survey

RANK	1957-58 Min.	1957-58 Max.	1958-59 Min.	1958-59 Max.
Instructor (only those holding Ph.D.)	43- 50	48- 50	50- 57	55- 60
Assistant Professor	53- 60	56- 70	55- 65	65- 77
Associate Professor	65- 80	75- 95	75- 83	80-100
Professor	80-110	95-190	90-120	115-200

OTHER STATE UNIVERSITIES AND COLLEGES

Number of questionnaires mailed: 16
Number of usable returns: 14

Total number on the staffs working full time on the campus

RANK	1957-58 (Actual)	1958-59 (Employed for the academic year as of now)
Instructor (only those holding Ph.D.)	11	11
Assistant Professor	73-1/2	82
Associate Professor	58	63
Professor	76-1/4	78-1/4
TOTAL	218-3/4	234-1/4

	Salary Survey			
	1957-58		1958-59	
RANK	Min.	Max.	Min.	Max.
Instructor (only those holding Ph.D.)	41-53	48- 53	45- 60	50- 60
Assistant Professor	49-65	49- 75	50- 66	55- 77
Associate Professor	52-80	64- 85	52- 82	52- 87
Professor	63-95	80-125	66- 95	85-130

LARGE NON-STATE UNIVERSITIES AND COLLEGES

Number of questionnaires mailed: 10
Number of usable returns: 8

Total number on the staffs working full time on the campus

RANK	1957-58 (Actual)	1958-59 (Employed for the academic year as of now)
Instructor (only those holding Ph.D.)	17	9
Assistant Professor	30	39
Associate Professor	56	57
Professor	53	59
TOTAL	156	155

	Salary Survey			
	1957-58		1958-59	
RANK	Min.	Max.	Min.	Max.
Instructor (only those holding Ph.D.)	48-50	48- 52	50- 52	50- 52
Assistant Professor	52-60	54- 83	50- 65	58- 94
Associate Professor	60-74	74- 97	58- 79	71-100
Professor	78-97	98-170	75-105	115-170

SMALLER NON-STATE UNIVERSITIES AND COLLEGES

Number of questionnaires mailed: 7
Number of usable returns: 3

Total number on the staffs working full time on the campus

RANK	1957-58 (Actual)	1958-59 (Employed for the academic year as of now)
Instructor (only those holding Ph.D.)	8	6
Assistant Professor	17-2/3	16
Associate Professor	8-1/3	8
Professor	12-1/2	12-1/2
TOTAL	46-1/2	42-1/2

Salary Survey

RANK	1957-58 Min.	1957-58 Max	1958-59 Min.	1958-59 Max.
Instructor (only those holding Ph.D.)	44-45	50- 55	55-55	55- 63
Assistant Professor	50-57	53- 65	52-60	53- 75
Associate Professor	58-70	60- 70	59-75	63- 76
Professor	75-90	87-120	75-95	90-130

INSTITUTES OF TECHNOLOGY

Number of questionnaires mailed: 5
Number of usable returns: 3

Total number on the staffs working full time on the campus

RANK	1957-58 (Actual)	1958-59 (Employed for the academic year as of now)
Instructor (only those holding Ph.D.)	15-1/2	14
Assistant Professor	18-1/2	21-1/2
Associate Professor	15	16
Professor	21	21
TOTAL	70	72-1/2

Salary Survey

RANK	1957-58 Min.	1957-58 Max.	1958-59 Min.	1958-59 Max.
Instructor (only those holding Ph.D.)	48-55	52- 60	50-60	53- 65
Assistant Professor	53-63	62- 80	55-70	60- 78
Associate Professor	48-75	80-105	58-80	72-115
Professor	54-96	105-150	64-83	108-160

INSTITUTIONAL MEMBERS OF THE SOCIETY NOT INCLUDED IN THE ABOVE TABULATIONS

Number of questionnaires mailed: 85
Number of usable returns: 52

Total number on the staffs working full time on the campus

RANK	1957-58 (Actual)	1958-59 (Employed for the academic year as of now)
Instructor (only those holding Ph.D.)	32	30
Assistant Professor	160-5/12	175-1/4
Associate Professor	124	131-1/2
Professor	125-1/3	131-2/3
TOTAL	441-3/4	468-5/12

Salary Survey

RANK	1957-58 Min.	1957-58 Max.	1958-59 Min.	1958-59 Max.
Instructor (only those holding Ph.D.)	32- 70	43- 78	38- 71	50- 81
Assistant Professor	37- 75	45- 95	40- 78	46- 95
Associate Professor	47- 98	54-109	53-102	55-109
Professor	56-112	64-139	56-117	69-154

PART II

DATA FROM
THE NATIONAL REGISTER OF SCIENTIFIC AND TECHNICAL PERSONNEL

Medians and Extremes of Annual Incomes* reported by 10,709 Respondents to the Questionnaires of the Mathematical Sciences Section of the National Register of Scientific and Technical Personnel.

Data from Register files as of May, 1958.

Highest degree held	Academic and non-profit Organizations	Industrial and business, except actuaries	Actuaries	Government	All other, including Military
Bachelor's (No. of responses)	(421)	(1,192)	(582)	(204)	(125)
Lowest group	Below 20	Below 20	Below 20	Below 20	Below 20
Median group	40-60	80-90	100-120	70-80	40-60
Highest group	200 +	200 +	200 +	160-200	200 +
Master's (No. of responses)	(2,058)	(1,522)	(320)	(500)	(175)
Lowest group	Below 20	Below 20	Below 20	Below 20	Below 20
Median group	40-60	90-100	100-120	80-90	60-70
Highest group	200 +	200 +	200 +	120-160	200 +
Doctoral (No. of responses)	(2,541)	(562)	(18)	(230)	(55)
Lowest group	Below 20	20-40	40-60	20-40	Below 20
Median group	70-80	100-120	120-160	90-100	90-100
Highest group	200 +	200 +	200 +	160-200	200 +
(Total no. of responses)	(5,020)	(3,276)	(920)	(1,138)	(355)

*The NRSTP questionnaire requests that income information be furnished only in terms of the following groups, which are identified here in terms of hundreds of dollars: Below 20, 20-40, 40-60, 60-70, 70-80, 80-90, 100-120, 120-160, 160-200, and 200 +. The upper end-point is excluded in each group.

PERSONAL ITEMS

(This section is restricted to members of the Society)

Assistant Professor W. G. Bade of the University of California, Berkeley, has been awarded a Senior Post-doctoral National Science Foundation Fellowship for study at Yale University for the academic year 1958-1959.

Assistant Professor Errett Bishop of the University of California, Berkeley, has been awarded a Sloan Foundation Fellowship, which will relieve him of half of his teaching duties for two years, 1958-1960.

Professor R. C. Buck, on leave from the University of Wisconsin, has received a Guggenheim Fellowship and will be at Stanford University for the academic year 1958-1959.

Professor H. G. Cohen of Rensselaer Polytechnic Institute will be a senior research scholar under the Fulbright program at the Technische Hogeschool, Delft, Netherlands, for the academic year 1958-1959.

Associate Professor John Dyer-Bennet has been awarded a National Science Foundation Science Faculty Fellowship and will be in Zurich, Switzerland, on leave from Purdue University, for the academic year 1958-1959.

Professor N. J. Fine of the University of Pennsylvania will be on sabbatical leave for the academic year 1958-1959 as a John Simon Guggenheim Memorial Fellow at the Institute for Advanced Study.

Professor D. T. Finkbeiner of Kenyon College has received a National Science Foundation Fellowship and will be at Princeton University for the academic year 1958-1959.

Associate Professor Leonard Gillman, on sabbatical leave from Purdue University, has received a Guggenheim Fellowship and will be at the Institute for Advanced Study for the coming year.

Dr. Seymour Goldberg of the University of California, Los Angeles, has been awarded a Charles Brown Fellowship to do research at Hebrew University, Jerusalem, Israel.

The National Science Foundation has announced a grant to Lehigh University to support studies by Professor Theodore Hailperin.

Professor V. L. Klee, Jr. of the University of Washington has received a National Science Foundation Senior Post-doctoral Fellowship and will be at the University of Copenhagen, Denmark, for the academic year 1958-1959.

Professor Morris Kline of New York University will be on sabbatical leave during the academic year 1958-1959. He will be at the Mathematisches Institut, Technische Hochschule, Aachen, Germany as a Fulbright lecturer and Guggenheim Fellow.

Mr. H. H. Martens of the Bell Telephone Laboratories has received a Bell Telephone Laboratories Communications Development Training Program Fellowship to complete the work required for a Ph. D. degree while continuing as a member of the technical staff of the Bell Laboratories.

Associate Professor Teruhisa Matsusaka of Northwestern University has been awarded a research grant by the National Science Foundation to conduct a study of the application of algebraic methods to geometry.

Dr. R. S. Palais of the University of Chicago has received a National Science Foundation Postdoctoral Fellowship and will be a member of the Institute for Advanced Study for the academic year 1958-1959.

Dr. P. M. Parsons of the applied physics laboratory of Johns Hopkins University, Silver Spring, has been awarded a William S. Parsons Fellowship at Johns Hopkins University, Baltimore, Maryland, for the academic year 1958-1959.

Assistant Professor P. E. Thomas of the University of California, Berkeley, has been awarded a Postdoctoral National Science Foundation Fellowship for study at Oxford University, Oxford, England, for the academic year 1958-1959.

Professor L. C. Young, on research leave in Europe from the University of Wisconsin, has received a George Ives Haight travelling fellowship.

Professor Oscar Zariski of Harvard University has been elected to a corresponding membership by the Academia Brasileira de Ciencias, Rio de Janeiro, Brazil.

Associate Professor S. A. Amitsur of the Hebrew University, Jerusalem, Israel has been appointed to a visiting professorship at the University of Notre Dame.

Dr. J. J. Andrews of the Union Carbide Nuclear Company has been appointed to an assistant professorship at the University of Georgia.

Dr. P. M. Anselone of the Johns Hopkins University will join the staff of the Mathematics Research Center of the University of Wisconsin.

Dr. E. F. Assmus, Jr. of Harvard University has been appointed a research associate at Columbia University.

Dean H. P. Atkins of the University of Rochester has been appointed to a professorship at the University of Richmond.

Dr. A. K. Aziz of the University of Maryland has been appointed to an assistant professorship at Georgetown University.

Dr. R. W. Bagley of the Lockheed Aircraft Corporation has been appointed to a professorship at Mississippi Southern College.

Assistant Professor O. R. E. Barker of the Hampton Institute has been appointed to an assistant professorship at the State Teacher

College, Trenton, New Jersey.

Assistant Professor W. A. Beck of Bucknell University has been appointed to an assistant professorship at Chatham College.

Mr. William Beck of the University of Southern California, Los Angeles, has been appointed a research engineer at the jet propulsion laboratory of the California Institute of Technology.

Mr. A. T. Bharucha-Reid is on leave from the University of Oregon at the Mathematical Institute of the University of Wroclaw, Wroclaw, Poland.

Dr. W. W. Boone of the University of Oslo, Oslo, Norway has been appointed to an associate professorship at the University of Illinois.

Dr. G. E. Bredon of Harvard University has been appointed a member of the Institute for Advanced Study.

Assistant Professor H. J. Bremermann of the University of Washington has been appointed to an associate professorship at the University of California, Berkeley. He will be on leave as a member of the Institute for Advanced Study.

Mr. T. S. Bright of Burns and Roe, Inc. has accepted a position as senior mechanical engineer with the Boeing Airplane Company, Seattle, Washington.

Associate Professor R. G. Buschman of the University of Wichita has been appointed to an assistant professorship at Oregon State College.

Assistant Professor R. C. Carson of Western Reserve University has accepted a position as development engineer with the Goodyear Aircraft Company, Akron, Ohio.

Assistant Professor C.-C. Chang of the University of Southern California has been appointed to an assistant professorship at the University of California, Los Angeles.

Dr. John Christopher of Burroughs Corporation has been appointed to an assistant professorship at Sacramento State College.

Mr. R. G. Clow of the University of California, Berkeley, has accepted a position as senior scientist with the Lockheed Aircraft Corporation, Palo Alto, California.

Dr. L. W. Cohen of the National Science Foundation, Washington, D. C. has been appointed to a professorship at the University of Maryland.

Dr. Geraldine A. Coon of the Taylor Instrument Company has been appointed to an assistant professorship at the University of Connecticut.

Assistant Professor H. O. Cordes of the University of Southern California has been appointed to an assistant professorship at the University of California, Berkeley.

Dr. H. H. Corson, on leave from Tulane University of Louisiana to accept an Office of Naval Research Post Doctoral Fellowship at the University of Washington, has been promoted to an assistant professorship.

Dr. R. H. Crowell of Massachusetts Institute of Technology has been appointed to an assistant professorship at Dartmouth College.

Dr. Chandler Davis of the Institute for Advanced Study has accepted a position as editorial consultant for the MATHEMATICAL REVIEWS of the American Mathematical Society, Providence, Rhode Island.

Assistant Professor H. F. Davis of the University of British Columbia has been appointed to an assistant professorship at the Royal Military College, Kingston, Ontario, Canada.

Assistant Professor J. C. E. Dekker of Northwestern University has been appointed to an associate professorship at the University of Kansas.

Associate Professor René De Vogelaere of the University of Notre Dame has been appointed an associate professor and an associate research mathematician in the Computer Center, University of California, Berkeley.

Mr. D. G. Dickson of Columbia University has been appointed to an assistant professorship at the University of Michigan.

Assistant Professor W. P. Eames of the University of New Brunswick, Canada, has been appointed a lecturer at Sir John Cass College, University of London, England.

Assistant Professor James Eells, Jr., on leave from the University of California, Berkeley, has been appointed to a visiting assistant professorship at Columbia University for the academic year 1958-1959.

Dr. H. G. Eggleston of Cambridge University has been appointed to a professorship at the University of London, London, England.

Associate Professor J. W. Ellis of Florida State University has been appointed to an associate professorship at Louisiana State University and Agricultural and Mechanical College.

Dr. M. H. M. Esser of the University of Maryland has been appointed to an assistant professorship at the University of Dayton.

Mr. H. L. Farris of Black, Sivalls and Bryson, Inc. has accepted a position as editorial assistant with the Society of Exploration Geophysicists, Tulsa, Oklahoma.

Associate Professor S. R. Fary of the University of Montreal, Canada, has been appointed to an associate professorship at the University of California, Berkeley.

Assistant Professor Chester Feldman of the University of New Hampshire has been appointed to an assistant professorship at the University of Connecticut.

Mr. C. T. Fike of the Oak Ridge Institute of Nuclear Studies has accepted a position with the Mathematics Panel of the Oak Ridge National Laboratory.

Dr. S. R. Foguel of New York University has been appointed to a visiting assistant professorship at the University of California, Berkeley, for the academic year 1958-1959.

Dr. D. J. Foulis of the University of Chicago has been appointed to an assistant professorship at Lehigh University.

Dr. Phyllis A. Fox of New York University has been appointed a research associate at Massachusetts Institute of Technology.

Mr. H. P. Friedman of the Reeves Instrument Corporation has accepted a position as senior mathematician with the Bulova Research and Development Laboratories, Woodside, New York.

Mr. K. S. Gale of the Armour Research Foundation, Tucson, Arizona has been appointed assistant director of the Numerical Analysis Laboratory of the University of Arizona.

Dr. R. A. Gambill of the United States Naval Avionics Facility, Indianapolis, Indiana has accepted a position as mathematician with the General Motors Corporation of Indianapolis.

Dr. Gladys Gibbens has retired from the University of Minnesota with the title, Associate Professor Emeritus.

Mr. Jesse Hamman of the University of Michigan has accepted a position as aeronautical research engineer with the National Advisory Committee for Aeronautics, Cleveland, Ohio.

Miss Nancy L. Hannye of Cornell University has been appointed to an assistant professorship at Michigan State University.

Dr. Robert Heller, Jr. of the University of Texas has been appointed to an assistant professorship at the University of Houston.

Assistant Professor L. L. Helms of Michigan State University has been appointed to an assistant professorship at the University of Illinois.

Professor G. P. Hochschild of the University of Illinois has been appointed to a professorship at the University of California, Berkeley.

Assistant Professor J. H. Hodges of the University of Buffalo has accepted a position as research mathematician with Cornell Aeronautical Laboratory, Inc., Buffalo, New York.

Assistant Professor W. M. Huebsch of the University of Notre Dame will be a member of the Institute for Advanced Study for the academic year 1958-1959.

Dr. G. A. Hufford of Stanford University has been appointed to an assistant professorship at the University of Washington.

Assistant Professor D. R. Hughes, on leave from Ohio State University for the academic year 1958-1959, has been appointed a visiting lecturer and research associate at the University of Chicago.

Mr. R. E. Hux of Chance Vought Aircraft, Inc., has accepted a position as engineer with the General Electric Company, Syracuse, New York.

Associate Professor L. K. Jackson, on leave from the University of Nebraska for one year, has been appointed to a visiting associate professorship at the University of Oregon.

Dean J. X. Jamrich of Doane College has been appointed to an associate professorship at Michigan State University.

Dr. H. B. Jenkins of the California Institute of Technology has been appointed a temporary member of the Institute of Mathematical Sciences, New York University.

Associate Professor Meyer Jerison is on leave from Purdue University as a temporary member of the Institute for Advanced Study.

Associate Professor Roberta F. Johnson of Wilson College has been appointed to an associate professorship at Colorado State University.

Associate Professor L. H. Kanter of the Drexel Institute of Technology has been appointed to an associate professorship at the New Jersey State Teachers College, Upper Montclair, New Jersey.

Mr. H. M. Kaplan of Johns Hopkins University has been appointed to an assistant professorship at the United States Naval Academy, Annapolis, Maryland.

Dr. R. B. Kelman of the University of Illinois has accepted a position as mathematician in the research center of the International Business Machines Corporation, Ossining, New York.

Dr. J. E. Kimber, Jr. of Tufts University has been appointed to an assistant professorship at the University of Nebraska.

Dr. Naoki Kimura of the Tokyo Institute of Technology and Tulane University of Louisiana has been appointed to an assistant professorship at the University of Washington.

Dr. M. I. Knopp of the University of Illinois has accepted a position as research mathematician with the Ramo-Wooldridge Corporation, Los Angeles, California.

Dr. George Kolettis, Jr. of Northwestern University has been appointed a research associate at the University of Notre Dame.

Assistant Professor Bertram Kostant of the University of California, Berkeley, has been appointed a member of the Miller Institute for Basic Research, University of California, for the academic year 1958-1959.

Dr. J. B. Kruskal of the University of Wisconsin has been appointed to an assistant professorship at the University of Michigan.

Associate Professor H. W. Kuhn, on leave from Bryn Mawr College, will be a visiting member of the London School of Economics, London, England for a year.

Mr. Ali Kyrala of Lessells and Associates, Inc., Boston, Massachusetts has accepted a position as mathematical physicist with the Goodyear Aircraft Corporation, Litchfield Park, Arizona.

Professor J. P. LaSalle, on leave from the University of Notre Dame, has accepted a position as mathematician with RIAS, Inc., a subsidiary of The Martin Company, Baltimore, Maryland.

Dr. Detlef Laugwitz of the Technische Hochschule, Munich, Germany has been appointed a Dozent at the Technische Hochschule, Darmstadt, Germany.

Professor D. H. Lehmer of the University of California, Berkeley, will be on sabbatical leave for the academic year 1958-1959. He has already begun a trip around the world.

Dr. H. S. Leonard, Jr. of Harvard University has been appointed to an assistant professorship at Carnegie Institute of Technology.

Mr. I. J. Lieberman of the Litton Industries has accepted a position as manager of engineering with the Philco Corporation, Philadelphia, Pennsylvania.

Dr. E. L. Lima of the University of Chicago has been appointed a research associate at the Instituto de Matematica Pura e Aplicada, Rio de Janeiro, Brazil.

Mr. C. R. Lindholm of Motorola, Inc., has accepted a position as member of the research staff of RAND Corporation, Santa Monica, California.

Assistant Professor W. D. Lindstrom of the Iowa State College of Agriculture and Mechanic Arts has been appointed to an associate professorship at Kenyon College.

Professor H. W. Linscheid of Southwestern State College has been appointed to an associate professorship at the University of Wichita.

Assistant Professor W. G. Lister of Brown University has been appointed to a professorship at the State University College on Long Island, Oyster Bay, New York.

Assistant Professor John McCarthy of Dartmouth College has been appointed to an assistant professorship at the Massachusetts Institute of Technology.

Dr. L. R. McMurray of the University of Illinois has accepted a position as senior research engineer with the autonetics division of North American Aviation, Inc., Downey, California.

Mr. J. C. Mairhuber of the University of Rochester has been appointed to an assistant professorship at the University of New Hampshire.

Associate Professor A. V. Martin, on leave from the University of New Mexico, has been appointed a visiting staff member of the University of Sheffield, Sheffield, England.

Dr. K. L. Miller of Brown University has accepted a position as member of the technical staff of the Ramo-Wooldridge Corporation, Los Angeles, California.

Associate Professor K. S. Miller of New York University will be on sabbatical leave at the Institute for Advanced Study during the academic year 1958-1959.

Dr. R. A. Moore of the University of Texas has accepted a position as mathematician in the personnel laboratory of the Lackland Air Force Base, Texas.

Professor Andrzej Mostowski of the University of Warsaw has been appointed a visiting professor at the University of California, Berkeley, for the academic year 1958-1959.

Dr. Leopoldo Nachbin of the Institute for Advanced Study has been appointed to a membership as lecturer at the Instituto de Matematica Pura e Aplicada, Rio de Janeiro, Brazil.

Dr. Anil Nerode of the University of Chicago has been appointed to a visiting assistant professorship at the University of California, Berkeley, for the academic year 1958-1959.

Dr. Hanna Neumann of The University, Hull, England has been appointed a lecturer at the Manchester College of Science and Technology, Manchester, England.

Mr. L. W. Neustadt of the Reeves Instrument Company has accepted a position as staff mathematician with the Space Technology Laboratories, Los Angeles, California.

Dr. E. A. Newburg of the University of Illinois has accepted a position as mathematician with Combustion Engineering, Inc., Windsor, Connecticut.

Dr. K. L. Nielsen of the Naval Avionics Facility, Indianapolis, Indiana has accepted a position as chief mathematician with the Allison Division of the General Motors Corporation of Indianapolis.

Associate Professor J. A. Nohel of Georgia Institute of Technology has been promoted to a professorship. He will be on leave for the academic year 1958-1959 at the mathematics research center of the University of Wisconsin.

Dr. R. J. Nunke of Yale University has been appointed to an assistant professorship at the University of Washington.

Dr. J. W. Odle of the University of Michigan has accepted a position as director of Airborne Defensive Systems, AVCO Manufacturing Corporation, Cincinnati, Ohio.

Assistant Professor Makoto Ohtsuka of Nagoya University, Nagoya, Japan has been appointed to a professorship at Hiroshima University.

Associate Professor Ingram Olkin of Michigan State University is on sabbatical leave at Stanford University.

Dr. O. T. O'Meara has been appointed to a visiting assistant professorship at Princeton University.

Associate Professor Anne F. O'Neill of Wheaton College, Norton, Massachusetts will be on sabbatical leave in England until January 1959.

Assistant Professor R. B. Paine of Central Michigan College has been appointed to an associate professorship at Austin State College.

Dr. C. D. Papakyriakopoulos of the Institute for Advanced Study has been appointed a research associate at Princeton University for the academic year 1958-1959.

Associate Professor G. A. Parkinson of the University of Wisconsin, Milwaukee, has been appointed director of the Milwaukee Vocational and Adult Schools.

Dr. M. H. Pearl of the National Bureau of Standards, Washington, D. C., has been appointed to an assistant professorship at the University of Maryland.

Mr. R. J. Pegis of the University of Toronto has accepted a position as mathematician with Bausch and Lomb Optical Company, Rochester, New York. He has also been appointed an assistant lecturer at the University of Rochester.

Professor Ruth M. Peters of St. Lawrence University has been appointed to an associate professorship at the University of New Hampshire.

Professor R. S. Phillips of the University of Southern California has been appointed to a professorship at the University of California, Los Angeles.

Dr. R. P. Polivka of the University of Illinois has accepted a position as associate engineer with the International Business Machines Corporation, Poughkeepsie, New York.

Dr. Barth Pollak of the Illinois Institute of Technology has been appointed to an assistant professorship at Syracuse University.

Professor Emeritus George Polya of Stanford University has been appointed a visiting professor at the University of California, Berkeley, for the spring term 1959.

Associate Professor L. E. Pursell, on leave from Grinnell College, has been appointed to a visiting assistant professorship at Purdue University for the academic year 1958-1959.

Acting Assistant Professor Ronald Pyke of Stanford University has been appointed to an assistant professorship at Columbia University.

Dr. F. A. Raymond of the University of Michigan has been appointed a member of the Institute for Advanced Study for the academic year 1958-1959.

Assistant Professor R. A. Roberts of the University of Miami, Coral Gables, Florida has accepted a position in the atomic power division of the Westinghouse Electric Corporation, Pittsburgh, Pennsylvania.

Associate Professor M. A. Rosenlicht of Northwestern University has been appointed to a professorship at the University of California, Berkeley.

Mr. David Rubinfien of the Illinois Institute of Technology has accepted a position as marketing services manager with the Raytheon Manufacturing Company, Needham Heights, Massachusetts.

Professor Walter Rudin of the University of Rochester has been appointed a research associate at Yale University for the academic year 1958-1959.

Dr. I. D. Ruggles of the Iowa State College has been appointed to an assistant professorship at San Jose State College.

Professor Henry Scheffé of the University of California, Berkeley, will be a visiting professor at Princeton University for the academic year 1958-1959, where his work with the Statistical Techniques Research Group will be partially supported by a grant from the National Science Foundation.

Professor E. V. Schenkman of Louisiana State University has been appointed to a visiting professorship at the University of Wisconsin.

Associate Professor Seymour Schuster of the Polytechnic Institute of Brooklyn has been appointed to a visiting associate professorship at Carleton College.

Assistant Professor Berthold Schweizer of the San Diego State College has been appointed to an assistant professorship at the University of California, Los Angeles.

Professor C. H. W. Sedgewick of the University of Connecticut has retired and has accepted a position as statistician at the Bureau of the Census, Washington, D. C.

Assistant Professor Esther Seiden, on leave from Northwestern University, will be at the Indian Statistical Institute, Calcutta, India for the academic year 1958-1959.

Associate Professor R. J. Silverman, on leave from the Illinois Institute of Technology, has been appointed to a visiting professorship at the University of Puerto Rico, Mayaguez, Puerto Rico.

Assistant Professor R. C. Simpson, Jr. of the United States Naval Academy has been appointed to an associate professorship at St. Lawrence University.

Associate Professor K. T. Smith, on leave from the University of Kansas, has been appointed to an associate professorship at the University of Wisconsin.

Assistant Professor M. B. Smith, Jr. of Florida State University has been appointed to an assistant professorship at the University of Utah.

Mr. W. E. Smith of the University of California, Los Angeles, has been appointed to an assistant professorship at Occidental College.

Dr. L. M. Sonneborn of Princeton University has been appointed to an assistant professorship at the University of Kansas.

Dr. E. M. Stein of Massachusetts Institute of Technology has been appointed to a position as lecturer at the University of Chicago.

Assistant Professor T. D. Sterling of Michigan State University has been appointed director of the Statistic Section in the Department of Preventive Medicine and assistant professor, Medical College, University of Cincinnati.

Associate Professor Choy-tak Taam of the Catholic University of America has been appointed to a professorship at Georgetown University.

Dr. R. B. Talmadge of the Lockheed Aircraft Corporation has accepted a position as computer specialist with the International Business Machines Corporation, Los Angeles, California.

Professor Alfred Tarski of the University of California, Berkeley, has been appointed a member of the Miller Institute for Basic Research, University of California, January 1958 through December 1960.

Mr. J. D. Thomas of the University of Oklahoma has accepted a position as staff member at the Los Alamos Scientific Laboratory of the University of California.

Assistant Professor C. J. Titus of the University of Michigan has been appointed to a visiting assistant professorship at the University of California, Berkeley.

Dr. H. F. Trotter of Princeton University has been appointed to an assistant professorship at Queen's University, Kingston, Ontario, Canada.

Assistant Professor R. L. Vaught of the University of Washington has been appointed to an assistant professorship at the University of California, Berkeley.

Associate Professor J. H. Wahab of Georgia Institute of Technology has been appointed to a professorship at Louisiana State University.

Mr. C. M. Warden of the University of Missouri has been appointed a research engineer at the jet propulsion laboratory of the California Institute of Technology.

Assistant Professor N. T. Watson of Clark University has accepted a position as engineer with Sylvania Electric Products Company, Waltham, Massachusetts.

Dr. R. L. Wilson of CONVAIR, Fort Worth, Texas has been appointed to a professorship at the Ohio Wesleyan University.

Mr. D. M. G. Wishart of the University of Aberdeen has been appointed a lecturer at the University of Birmingham, Birmingham, England.

Assistant Professor J. W. Woll, Jr. has been appointed to an assistant professorship at the University of California, Berkeley.

Professor Yung-Chow Wong of the University of Hong Kong, on leave for the academic year 1958-1959, will be at the Institute for Advanced Study for the first semester, and at the University of Chicago for the second semester.

Dr. J. N. Younglove of the University of Texas has been appointed to an assistant professorship at the University of Missouri.

The following promotions are announced:

W. H. Benson, Dickinson College, to an associate professorship.

W. F. Darsow, DePaul University, to an assistant professorship.

Harley Flanders, University of California, Berkeley, to an associate professorship.

R. P. Gosselin, University of Connecticut, to an associate professorship.

D. L. Guy, University of Nebraska, to an assistant professorship.

Henry Helson, University of California, Berkeley, to an associate professorship.

Leon Henkin, University of California, Berkeley, to a professorship.

W. B. Houston, Jr., Morehouse College, to an associate professorship.

H. D. Huskey, University of California, Berkeley, to a professorship.

J. R. Jackson, University of California, Los Angeles, to principal investigator and project director, Management Sciences Research Project, and associate professor of Business Administration.

E. P. Merkes, DePaul University, to an assistant professorship.

M. H. Protter, University of California, Berkeley, to a professorship.

Edgar Reich, University of Minnesota, to an associate professorship.

Abraham Seidenberg, University of California, Berkeley, to a professorship.

Selmo Tauber, University of Kansas, to an assistant professorship.

L. O. Thompson, Marshall College, to an assistant professorship.

W. R. Utz, Jr., University of Missouri, to a professorship.

G. L. Weiss, DePaul University, to an assistant professorship.

Harold Widom, Cornell University, to an assistant professorship.

The following appointments to instructorships are announced:

Aarhus University, Denmark: Mr. L. C. Mejlbo; University of California, Berkeley: Dr. Charles Ballantine, Dr. C. W. Clark, Dr. Denis Rutovitz, Dr. Roy Takenaga; University of Chicago: Dr. D. S. Scott; Cornell University: Dr. William Browder; Duke University: Dr. G. H. Meisters; Harvard University: Dr. Hugh Gordon; Illinois Institute of Technology: Mr. E. J. Voltaggio; International Christian University, Tokyo, Japan: Dr. R. H. Geeslin; University of Maryland: Dr. G. R. Lehner; University of Massachusetts: Dr. Ward Barnes; University of Nebraska: Mr. J. M. Anderson; University of North Carolina: Dr. J. H. Wells; Northwestern University: Dr. Eben Matlis; Princeton University: Dr. Harry Kesten; Stanford University: Mr. W. W. Tait; Tulane University of Louisiana: Dr. L. B. Treybig; United States Naval Academy: Lt. Commander A. A. Folop; Wellesley College: Dr. Dorothy J. Christensen; University of Wisconsin: Dr. G. O. Losey.

Deaths:

Dr. Milton Abramowitz of the National Bureau of Standards, Washington, D. C. died on July 5, 1958 at the age of 43 years. He had been a member of the Society for 17 years.

Professor Emeritus W. F. Durand of Stanford University died on August 9, 1958 at the age of 99 years. He had been a member of the Society for 34 years.

Dr. D. A. Flanders, director of the applied mathematics division of the Argonne National Laboratory, Lemont, Illinois, died on June 27, 1958 at the age of 57 years. He had been a member of the Society for 34 years.

Assistant Professor R. E. Luce of Rutgers, The State University, died on January 20, 1958 at the age of 46 years.

Professor František Vyčichlo of the Charles University, Prague, Czechoslavakia died on January 6, 1958 at the age of 42 years.

Dr. Marion B. White, a former teacher at Carleton College, died on January 30, 1958 at age of eighty-six years. She had been a member of the Society for 48 years.

LETTERS TO THE EDITOR

Editor, the NOTICES.

Authors can increase the usefulness of their bibliographies by including after each reference to an article a reference to a review of the article. For it is to the review that one turns first in following up a reference. This can be done in very short space by, for example, [7, 213], meaning volume 7, page 213 of the Mathematical Reviews; a note to this effect appearing at the top of the list. I have done this frequently, but find on my latest galley proofs that one of our American Journals has deleted these references.

Albert Wilansky

Editor, the NOTICES.

Although the National Science Foundation is well able to defend itself, perhaps it is worth indicating that at least one outsider thinks NSF is not as black as Professor Halmos (NOTICES, August, 1958) paints it.

Two features of NSF activity seem to be in question: the regulations governing fellowships and the administration of these regulations. As to the first, I agree with Professor Halmos: a minimum of supervision and red tape would tend to give better long term results than can be expected under the present arrangement, provided due care is exercised in making awards; the regulations should therefore be eliminated or weakened as much as possible. But I believe that Professor Halmos' letter confuses these two features, and I should like to emphasize their disjointness by discussing the latter.

I believe it is wrong to condemn NSF for acting in accordance with the regulations as presently written. Every NSF fellow knows, or should know, what these regulations are, and presumably has accepted them along with the fellowship. He knows that permission must be requested from NSF for major changes of program and for changes of locality, and he is not acting within the spirit of his fellowship agreement if he merely notifies NSF of his "proposed whereabouts as a matter of courtesy," and, incidentally, almost as a fait accompli.

It seems to me that the people at NSF are as sensible, intelligent, and sympathetic a group of bureaucrats as one can find in the city of Washington, and that they act as wisely and generously as can be hoped, within the framework of existing regulations. I do not believe that Dr. Dees' apparent misconceptions regarding vacations and proprietorship of the time between academic years can be held

mostly to blame in this particular instance. Rather, the major fault appears to lie with those who suited their actions to the regulations as they would have liked them to be, not as they are.

T. C. Holyoke
Science Faculty Fellow, NSF

MEMORANDA TO MEMBERS

NEW ABSTRACT BLANKS

The Society's abstract form has been slightly revised to bring it up to date. A supply of the new forms has been sent to each of the Associate Secretaries, from whom copies may be obtained on request. Abstract blanks may also be obtained from the Headquarters Offices, as in the past. Abstracts on the old forms are of course still acceptable.

STEPS TAKEN IN RECOGNITION OF THE REFERENCE VALUE OF THE NOTICES

Now that the abstracts of the papers presented at the meetings are appearing in the NOTICES, this journal will undoubtedly be regarded by many members of the Society as having a certain reference value which it did not formerly have. Some members may be reluctant to risk damage to or loss of their copies by taking them to the meetings.

The abstracts were transferred to the NOTICES mainly so that the abstracts of the papers presented in person could be published in advance of the meetings. It was hoped that this new service would increase the value of the NOTICES not only before the meetings but also at the meetings.

Members who hesitate to bring their subscription copies of the NOTICES to the meetings may be interested in the following steps which have been taken in recognition of the less transitory nature of the new contents:

1. Copies of the NOTICES are for sale at the registration desks of the meetings at the special low price of $1 per copy. Also, duplicates of the program sections are being distributed free at the meetings. Thus it is not really necessary to take one's subscription copy to a meeting.

2. There will be a substantial back-number sales stock of all issues from 1958 on. Thus libraries and individuals can easily replace any issues which are damaged or lost at meetings.

The back number prices, which were first published in the NOTICES for June, 1958, are as follows:

Complete volumes, seven issues	
List price	$14.00
Agents' price	11.90
Members' price	10.50

Single issues	
List price	$ 2.00
Agents' price	1.70
Members' price	1.50

Agents' discount is allowed only for material mailed directly from the Society to foreign addresses.

Incidentally, an index of the abstracts will be published in the December issue each year. It will be alphabetized by author, and unlike the indexes of abstracts which used to appear in the BULLETIN, it will give titles as well as page numbers of the abstracts.

CATALOGUE OF LECTURE NOTES

Supplement No. 3

UNIVERSITY OF CALIFORNIA, BERKELEY

The following item may be ordered from: Associated Students Store, University of California, Berkeley 4, California. This item is in mimeograph form.

TOSIO KATO, "Quadratic forms in Hilbert spaces and asymptotic perturbation series", Technical Report No. 7, Army Ordnance Project DA-04-200-ORD-171, Task Order 5. April 1955 $1.00

HARVARD UNIVERSITY

The following item announced on page 448 of the August, 1958, NOTICES as no longer available has been reprinted and is now available.

P. C. ROSENBLOOM, "Linear partial differential equations". (Advance payment only) $2.50

The above item may be ordered from: Department of Mathematics, Harvard University, 2 Divinity Avenue, Cambridge 38, Massachusetts.

NEW PUBLICATIONS

Ackeret, J. See Eulerus, L.

Aleksandrov, P. S. Combinatorial topology. Vol. 2. The Betti groups. Rochester, Graylock, 1957. 11 + 244 pp. $6.50.

Batchelor, G. K. See Taylor, G. I.

Bellemans, A. See Prigogine, I.

Bethe, H. A., and Salpeter, E. E. Quantum mechanics of one- and two-electron atoms. Berlin, Springer, 1957. 8 + 369 pp. $10.00.

Bianchi, L. Opere. Vol. VII. Problemi di Rotolamento. Ed. by the Unione Matematica Italiana with contributions from the Consiglio Nazionale delle Ricerche. Rome, Cremonese, 1957. 361 pp. 3500 Lire.

Bizadse, W. A. Zum Problem der Gleichungen vom gemischten Typus. (Mathematische Forschungsberichte, V.) Berlin, VEB Deutscher Verlag der Wissenschaften, 1957. 59 pp. 13.20 DM.

Boas, R. P. Jr., and Buck, R. C. Polynomial expansions of analytic functions. (Ergebnisse der Mathematik und ihrer Grenzgebiete. New Series, No. 19.) Berlin, Springer, 1958. 8 + 77 pp. 19.80 DM.

Bouligand, G. See Felix, L.

Bowman, E. H., and Fetter, R. B. Analysis for production management. Homewood, Irwin, 1957. 13 + 503 pp. $7.00.

Browne, E. T. Introduction to the theory of determinants and matrices. Chapel Hill, University of North Carolina Press, 1958. 11 + 270 pp. $7.50.

Buck, R. C. See Boas, R. P. Jr.

Cherubino, S. Calcolo delle matrici. (Consiglio Nazionale delle Ricerche: Monografie Matematiche, IV.) Rome, Cremonese, 1957. 7 + 322 pp. 4000 Lire.

Cogan, E. J., and Norman, R. Z. Handbook of calculus, difference and differential equations. (Prentice-Hall Mathematics Series.) Englewood Cliffs, Prentice-Hall, 1958. 12 + 263 pp. $4.50.

Comolet, R. Écoulement d'un fluide entre deux plans parallèles. Contribution à l'étude des butées d'air. (Publ. Sci. Tech. Ministère de l'Air, no. 334.) Paris, 1957. 4 + 68 pp., 15 plates. 1100 fr.

Conrady, A. E. Applied optics and optical design. New York, Dover, 1957. 5 + 518 pp. $2.95.

Constant, F. W. Theoretical physics: Thermodynamics, electromagnetism, waves, and particles. (Principles of Physics Series.) Reading, Addison-Wesley, 1958. 13 + 364 pp. $7.50.

Corazza, H. A. See Sakatow, P. S.

Cosgriff, R. L. Nonlinear control systems. (McGraw-Hill Series in Control Systems Engineering.) New York, McGraw-Hill, 1958. 8 + 328 pp. $9.00.

Costa de Beauregard, O. Théorie synthétique de la relativité restreinte et des quanta. (Les grands problèmes des sciences, VIII.) Preface by M. Lévy. Paris, Gauthier-Villars, 1957. 12 + 200 pp.

Čuvikov, N. T. Preobrazovanie ortogonal'nyh proekcii. Moscow, Gosudarstv. Izdat. "Sovetskaya Nauka". 1957. 176 pp. 4.30 rubles.

Day, M. M. Normed linear spaces. (Ergebnisse der Mathematik und ihrer Grenzbegiete. New Series, No. 21.) Berlin, Springer, 1958. 4 + 139 pp. 28.00 D.M.

Dienes, P. The Taylor series: an introduction to the theory of functions of a complex variable. New York, Dover, 1957. 10 + 552 pp. $2.75.

Dubarle, Le R. P. (H. D.) Initiation à la logique. (Collection de Logique Mathématique, Série A, XIII.) Paris, Gauthier-Villars, 1957. 90 pp. 1400 fr.

DuBois, P. H. Multivariate correlational analysis. New York, Harper, 1957. 15 + 202 pp., 2 plates. $4.50.

Easthope, C. E. Three dimensional dynamics. A vectorial treatment. New York, Academic Press, 1958. 8 + 277 pp. $7.80.

Eggleston, H. G. Convexity. (Cambridge Tracts in Mathematics and Mathematical Physics, No. 47.) New York, Cambridge University Press, 1958. 8 + 136 pp. $4.00.

Eggleston, H. G. Problems in Euclidean space: Application of convexity. (International Series of Monographs on Pure and Applied Mathematics, Vol. V.) New York, Pergamon, 1957. 8 + 165 pp $6.50.

Èl'sgol'c, L. È. Differencial'nye uravneniya. Moscow, Gosudarstv. Izdat. Tehn.-Teor. Lit., 1957. 272 pp. 5.70 rubles.

Eulerus, L. Opera omnia. Series 2. Opera mechanica et astronomica. Vol. XV. Commentationes mechanicae ad theoriam machinarum pertinentes. Vol. 1. Ed. by J. Ackeret. Lausanne, Societas Scientiarum Naturalium Helveticae, 1957. 41 + 318 pp.

Eves, H., and Newsom, C. V. An introduction to the foundations and fundamental concepts of mathematics. New York, Rinehart, 1958. 15 + 363 pp. $6.75.

Felix, L. L'aspect moderne des mathématiques. With a preface and commentary by G. Bouligand. Paris, Blanchard, 1957. 1 + 169 pp. 800 fr.

Fetter, R. B. See Bowman, E. H.

Finzi, B., and Udeschini, P. Esercizi di meccanica razionale. 3d enlarged ed. Milan, Tamburini, 1958. 8 + 546 pp.

Förstner, K., and Henn, R. Dynamische Produktions-Theorie und Lineare Programmierung. With a forword by W. Waffenschmidt. (Schriften zur wirtschaftswissenschaftlichen Forschung, Vol. 5.) Meisenheim-Glan, Hain, 1957. 125 pp. 14.50 DM.

Friedman, M. D. See Khinchin, A. I.

Fubini, G. Opere scelte. Vol. I. Ed by the Unione Matematica Italiana with contributions from the Consiglio Nazionale delle Ricerche. Rome, Cremonese, 1957. 369 pp., 1 plate. 3500 Lire.

Gatewood, B. E. Thermal stresses. New York, McGraw-Hill, 1957. 11 + 232 pp. $7.50.

Geiringer, H. See von Mises, R.

Gill, S. See Wilkes, M. V.

Gnedenko, B. V., and Hinčin, A. Ya. Elmentarnoe vvedenie v teoriyu veroyatnostei. 4th ed. Moscow, Gosudarstv. Izdat. Tehn.-Teor. Lit., 1957. 144 pp. 2.20 rubles.

Gotlieb, C. C., and Hume, J. N. P. High-speed data processing. (McGraw-Hill Series in Information Processing and Computers.) New York, McGraw-Hill, 1958. 11 + 338 pp. $9.50.

Günter, N. M. Die Potentialtheorie und ihre Anwendung auf Grundaufgaben der mathematischen Physik. Leipzig, Teubner, 1957. 10 + 342 pp. 18.00 DM.

Hartman, S., and Mikusiński, J. Teoria miary i całki Lebesgue'a. [Theory of measure and Lebesgue integral.] Warsaw, Panstwowe Wydawnictwo Naukowe, 1957. 140 pp. 10 zł.

Henn, R. See Förstner, K.

Hinčin, A. Ya. See Gnedenko, B. V.

Hirsch, R. Installation de la portance sur un profil muni d'une fente de soufflage au bord de fuite et correction due à l'envergure finie. (Publ. Sci. Tech. Ministère de l'Air, Notes Tech. no. 69.) Paris, 1957. 2 + 75 pp. 1000 fr.

Hobson, E. W. A treatise on plane and advanced trigonometry. 7th ed. New York, Dover, 1957. 15 + 383 pp. $1.95.

Hobson, E. W. The theory of functions of a real variable and the theory of Fourier's series. New York, Dover, 1958. Vol. I, 15 + 736 pp., $3.00. Vol. II, 10 + 780 pp. $3.00.

Hovanskiĭ, G. S. Nomogrammy s orientirovannym transparantom. Biblioteka prikladnogo analiza i vyčislitel'noĭ matematiki. Moscow. Gosudarstv. Izdat. Tehn.-Teor. Lit., 1957. 204 pp. 4.60 rubles.

Hume, J. N. P. See Gotlieb, C. C.

Kendall, M. G. A course in multivariate analysis. (Griffin's Statistical Monographs and Courses, no. 2.) New York, Hafner, 1957. 185 pp.

Khinchin, A. I. Mathematical foundations of information theory. Trans. by R. A. Silverman and M. D. Friedman. New York, Dover, 1957. 2 + 120 pp. $1.35.

Kochendörffer, R. Determinanten und Matrizen. (Mathematisch-Naturwissenschaftliche Bibliothek, 12.) Leipzig, Teubner, 1957. 6 + 144 pp. 6.60 DM.

Kreyszig, E. Differentialgeometrie. (Mathematik und ihre Anwendungen in Physik und Technik, Series A, Vol. 25.) Leipzig, Geest and Portig, 1957. 11 + 421 pp. 36.00 DM.

Lacey, W. N., and Sage, B. H. Thermodynamics of one-component systems. New York, Academic Press, 1957. 11 + 376 pp. $8.00.

Lappo-Danilevskiĭ, I. A. Primenenie funkciĭ ot matric k teorii lineĭnyh sistem obyknovennyh differencial'nyh uravneniĭ. Ed. by V. I. Smirnov with an introductory essay. Moscow, Gosudarstv. Izdat. Tehn.-Teor. Lit., 1957. 456 pp. 18.75 rubles.

Lévy, M. See Csta de Beauregard, O.

Le Lionnais, F. L'imitation de la pensée créatrice par les machines. Université de Paris, 1957. 31 pp.

van Lint, J. H. Hecke operators and Euler products. Leiden, Drukkerij "Luctor et Emergo", 1957. 51 pp., 1 insert.

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ABSTRACTS OF CONTRIBUTED PAPERS

SUMMER MEETING IN CAMBRIDGE, MASSACHUSETTS

August 26-29, 1958

548-109. Smbat Abian and A. B. Brown: On the solution of an implicit equation.

A method is given for finding the solution $y = Y(x) \equiv Y(x_1,\ldots,x_n)$ of an implicit equation $g(x;y)$ $g(x_1,x_2,\ldots,x_n;y) = 0$ by successive substitutions, under the hypotheses that g is continuous and real-valued in an open set $N \subset E^{n+1}$ containing the segment $x_i = a_i$, $i = 1,\ldots,n, |y - b| \leqq h$, with $h > 0$, and that there exist two constants C and D with $0 < C \leqq D$, such that $|g(a;b)| < Ch$, and that, for every pair of points (x;u), (x;v) in N with $u \neq v$, $C \leqq (g(x;u) - g(x;v))/(u - v) \leqq D$. Four appraisals of the remainder error at the mth stage of approximation, and a method of minimizing one of the appraisals, are given. The remaining three appraisals are valid regardless of errors made at earlier stages of the computation. It is also proved that if g(x;y) satisfies a Lipschitz condition in a subset of the x_i's, the solution will also satisfy a Lipschitz condition in the same x_i's. (Received July 7, 1958.)

548-110. Rafael Artzy: On automorphic-inverse properties in loops.

It is well known that (i) I.P. loops, all of whose isotopes are I.P., are Moufang, (ii) C.I. loops (satisfying the identity $xy \cdot x^J = y$, where x^J is the right inverse of x), all of whose isotopes are C.I., are abelian groups. The I.P. and C.I. identities of the isotopes can be replaced by weaker identities involving the mapping J: (i) I.P. loops, in all of whose isotopes J is an antiautomorphism, are Moufang, (ii) C.I. loops, in all of whose isotopes J is an automorphism, are abelian groups. (Received July 1, 1958.)

548-111. A. K. Aziz: On a higher order boundary value problem for hyperbolic partial differential equations.

Consider the normal form of the linear partial differential equation of the hyperbolic type with variable coefficients in three dimensions: (1) $u_{xyz} + au_{xy} + bu_{xz} + cu_{yz} + du_x + eu_y + fu_z + gu = h(x,y,z)$. It will be shown here that for a certain subclass of equations of this type a unique solution exists by prescribing merely three partial derivatives of u(x,y,z), one on each of the ad-

jacent faces of a parallelepiped, i.e. by prescribing $(\partial^m u/\partial x^m)(0,y,z)$, $(\partial^n u/\partial y^n)(x,0,z)$, $(\partial^p u/\partial z^p)(x,y,0)$, where m, n and p are any non-negative integers, instead of giving u(0,x,z), u(x,0,z) and u(x,y,0) as in the classical boundary value problem. The present result is obtained essentially by reducing this new problem to the case $m = n = p = 0$. (Received July 10, 1958.)

548-112. J. W. Blattner and L. J. Paige: An identity for Jordan algebras. Preliminary report.

Let J be a Jordan algebra and define the Jordan triple product $\{abc\} = (ab)c + (bc)a - (ca)b$. The identity $\{\{aba\}\, c\{aba\}\} = \{a\{b\{aca\}b\}a\}$ is known to be valid in special Jordan algebras. The identity has now been proved for the free Jordan algebra with three generators a,b,c, and, hence, the identity is valid in all Jordan algebras. The method of proof extends that used by M. Hall in establishing the identity $\{aba\}\{aba\} = \{a\{ba^2b\}a\}$. A partial basis of the free Jordan algebra with three generators a,b,c is constructed for all elements of total degree 7 (degree 4 in a, degree 2 in b, degree 1 in c.) Now expressing the terms arising from the Jordan triple products in terms of the partial basis, the identity is verified for the free Jordan algebra. (Received June 30, 1958.)

548-113. F. G. Brauer: Some results on uniqueness and successive approximations.

Most of the well-known theorems on uniqueness and convergence of successive approximations for the differential equation $x' = f(t,x)$ involve a single bound on the expression $|f(t,x_1) - f(t,x_2)|$. There is, however, a result of Krein and Krasnosel'skiĭ (Uspehi Mat. Nauk. vol. 11 (1956) pp. 209-213) and Luxemburg (Canad. J. Math. vol. 1 (1958) pp. 9-20) that if this expression is at the same time no greater than $C|x_1 - x_2|^\alpha$ and $k|x_1 - x_2|/t$, where C,α,k are constants with $0 < \alpha < 1$ and $k(1 - \alpha) < 1$, then the equation has a unique solution to which the successive approximations converge uniformly. Here, a generalization of this result is obtained, with a pair of conditions being imposed. It is assumed that $|f(t,x_1) - f(t,x_2)| \leqq \psi_i(t,|x_1 - x_2|)$, $i = 1,2$, where all solutions r(t) of $r' = \psi_1(t,r)$, $r(0) = 0$ obey $r(t) \leqq A(t)$ and the only solution s(t) of $s' = \psi_2(t,s)$ such that $\lim_{t\to 0} s(t)/B(t) = 0$ is the trivial solution. If A(t),B(t) are continuous functions with $A(0) = B(0) = 0$ and $\lim_{t\to 0} A(t)/B(t) = 0$, then the equation $x' = f(t,x)$ has a unique solution to which the successive approximations converge uniformly. (Received June 27, 1958.)

548-114. R. K. Brown: Typically-real functions.

The author proves that if $f(z) = z + a_2z^2 + \ldots, a_n$ real, is typically-real of order p (p a positive integer) in $|z| \leqq 1$ then f(z) is typically-real of order one in the circle of radius $R_p(a_1,\ldots,a_p)$. A Stieltjes-integral representation of the form $(2/\pi)\int_0^{\pi} P(a_1,a_2,\ldots,a_p,z,\nu,\phi)\,d\alpha(\phi)$ is obtained for f(z) and it is proved that if the real roots of $\partial P/\partial z$ are $w_i(a_1,a_2,\ldots,a_p,\nu,\phi)$, $i = 1,2,\ldots,k$, then $R_p(a_1,a_2,\ldots,a_p) = \min_{\nu,\phi}|w_i|$ where the minimum is taken over all $0 \leqq \nu \leqq \pi$, $0 \leqq \phi \leqq \pi$. The bound R_p is shown to be sharp over the class of all typically-real functions normalized as above in the sense that for $p = 2$ there exists a function which is typically-real of order two in the unit circle and is not typically-real in any circle of radius greater than R_2. (Received June 16, 1958.)

548-115. P. T. Church: A Riemann mapping theorem for quasi-conformal maps on certain domains in E^3.

One might ask: Is there any class of homeomorphisms in E^3 for which there is an analog of the Riemann Mapping Theorem? Since the Liouville Theorem on conformal homeomorphisms in E^3 rules out this class, the quasi-conformal homeomorphisms are suggested. (Let f, mapping an open set U of E^n into E^n, be a local homeomorphism with continuous first partial derivatives. f is quasi-conformal if there exists a real number $B > 0$ such that, if d_1 and d_2 are directional derivatives in any two directions, then $d_1/d_2 \leqq B$.) The author has the following partial answer to the above question: If C in E^3 is a topological (open) 3-cell whose boundary is a smooth topological sphere (i.e., at each point of the sphere there is a tangent plane, which turns continuously), then there is a quasi-conformal homeomorphism of the open unit ball onto C. (Received July 10, 1958.)

548-116. W. H. Cockcroft: The cohomology groups of certain fibre spaces.

Let (E,B,F,p) be a fibre space in the sense of Serre with B arcwise connected and $\pi_1(B)$ operating trivially on F. Let K be a commutative ring with unit element. Assume $H^*(F;K)$ (singular cohomology) to be the graded tensor product of monogenic (possibly truncated) polynomial algebras with transgressive generators (x_i) and to be of finite type. Let $x = x_{i_1}^{r_1} x_{i_2}^{r_2} \ldots x_{i_n}^{r_n}$, $i_1 < i_2 < \ldots < i_n$ be a typical basis element of $H^*(F,K)$. Let y_i be a cocycle in the transgression of x_i, with c_i the transgression co-chain such that $\partial_E c_i = p^* y_i$. Define coboundary operator ∂_L in $L = C^*(B;K) \otimes H^*(F;K)$ by setting $\partial(b \otimes x) = \partial_B b \otimes x +$

$(-1)^p(b \otimes 1)\partial(1 \otimes x)$ and (inductively) $\partial(1 \otimes x_i) = y_i \otimes 1, \partial(1 \otimes xx_i) = \partial(1 \otimes x)$ $\cdot(1 \otimes x_i) + (-1)^{qr}(y_i \otimes 1)(1 \otimes x) + (-1)^{r+1}\partial(1 \otimes x) \circ (y_i \otimes 1)$ ($i \geq i_n, xx_i \in$ basis, $(b \otimes x) \circ (b' \otimes x') = (-1)^{q(p'-1)}b \cup_1 b' \otimes xx'$, degrees $b,b',x,x_i = p,p',q,r$). Define an additive homomorphism λ: $L \to C^*(E;K)$ by setting $\lambda(b \otimes x) = p^*b \cup \lambda(1 \otimes x)$ and (inductively) $\lambda(1 \otimes x_i) = c_i$, $\lambda(1 \otimes xx_i) = \lambda(1 \otimes x) \cup c_i + (-1)^{r+1}\lambda(1 \otimes x) \cup_1 dc_i$. Theorem. λ induces an additive graded isomorphism $H^*(L,\partial_L) \approx H^*(E;K)$. Proof. As in A. Borel (Theorem 24.1, Ann. of Math. vol. 57 (1953) pp. 115-207) with obvious changes in view of singular cohomology and inductive definitions of ∂ and λ. (Received June 30, 1958.)

548-117. Eckford Cohen: The characteristic divisors of a polynomial function of a matrix.

Let A be a square matrix in an arbitrary field F and suppose f(x) to be an arbitrary polynomial of F[x]. The author gives a new proof of McCoy's results (Amer. J. Math. vol. 57 (1935) pp. 491-502) on the characteristic divisors of f(A). The proof is based on the decomposition theory of modules. (Received May 26, 1958.)

548-118. Eckford Cohen: A class of residue systems (mod r) and related arithmetical functions, II. Higher dimensional analogues.

Let P and Q be a conjugate pair of direct factor sets (Notices Amer. Math. Soc. April, 1958, Abstract 542-18). Arithmetical functions generalizing $\sigma(r)$ and $\phi(r)$ are defined: $\sigma_{\alpha,P}(r) = \sum_{d|r} d^\alpha$ $(r/d \in P)$, $\phi_{\alpha,P}(r) = \sum_{d|r} d^\alpha \mu_P(r/d)$. In case α is a positive integer t, it is shown that $\phi_{\alpha,P}(r) = J_{t,P}(r)$ where $J_{t,P}(r)$ denotes the number of integral vectors $\{a_1,\ldots,a_t\}$, $a_i \pmod r$, such that $((a_1,\ldots,a_t),r) \in P$. Mean value estimates for $\sigma_{\alpha,P}(r)$ and $\phi_{\alpha,P}(r)$ are obtained, greatly generalizing known results for $\sigma(r)$ and $\phi(r)$. The results obtained involve the higher zeta-functions $\zeta_P(s) = \sum_{n \in P} n^{-s}$, $\zeta_P^{-1}(s) = \sum_1^\infty \mu_Q(n)n^{-s}$, $s > 1$. Application is made to the determination of the density of t-dimensional sets satisfying $(a_1,\ldots,a_t) \in P$; a number of special cases are considered. The second Möbius inversion formula is extended to P-sets and applied to give a wide generalization of Meissel's formula $\sum_1^n \mu(m)[n/m] = 1$. As an application, the density of the k-free integers prime to a fixed number n is determined. (Received June 27, 1958.)

548-119. Eckford Cohen: The Brauer-Rademacher identity.

Let n and r be integers, $r > 0$, and let $\phi(r)$ and $\mu(r)$ denote, respectively, the Euler and Möbius functions. This paper contains a new proof of the following identity due to A. Brauer and H. Rademacher (Jahres. Deutsche Math. vol. 35 (1926) pp. 94-95): $\phi(r) \sum_{d|r,(d,n)=1} (d/\phi(d))\mu(r/d) = \mu(r) \sum_{d|(n,r)} d\mu(r/d)$. The proof is based on an arithmetical inversion formula in the theory of even functions (mod r). (Received June 30, 1958.)

548-120. W. E. Deskins: An extension of a theorem of Herstein.

Recently [Proc. Amer. Math. Soc. vol. 9 (1958) pp. 255-257] I. N. Herstein proved that a finite group containing a maximal subgroup which is abelian is solvable. We extend this to the following Theorem. If the finite group G contains a maximal subgroup which is nilpotent of class 2, then G contains normal subgroups H and K, $H \supset K$, such that (i) G/H and K are nilpotent groups of class 2 or less, and (ii) H/K is an elementary abelian p-group. Also we note that while Herstein's result is contained in B.Huppert's Satz 2 [Math. Z. vol. 60 (1954) pp. 409-434] ours is not. (Received July 7, 1958.)

548-121. M. J. Etter: On idempotent stochastic matrices. Preliminary report.

Let M denote the semigroup of $n \times n$ matrices with non-negative real entries and column sums equal to one. Let P denote the set of stochastic column vectors. If e is an idempotent element of M, then f is in H(e) (the maximal subgroup of M containing e) if and only if f is a permutation matrix (i.e. f permutes the extreme points of e(P)). If E is the set of idempotents of M, then E has at least n components. If x and y are two distinct extreme points of e(P), then x and y are orthogonal. Consequently, if an idempotent is in the boundary of M (relative to $n^2 - n$ space), then e has a zero in every column. Also e is in the interior of M if and only if e has no zero entries and all columns are identical. (Received July 3, 1958.)

548-122. D. G. Bourgin: Maps of spheres.

On combining the result [Bourgin, Rend. Mat. e Appl. vol. 15 (1956) pp. 178-189] that a certain index cannot decrease under symmetric maps, with results of Borel on heights of some generating elements in the cohomology ring of a homogeneous space of the Stieffel manifold modulo a prime power cyclic group

there results, for example, Theorem: Let $\{x\}$ be the smallest integer $\geq x$ and let $e(n) = 0$ for n odd and $= 1$ for n even. If k is an odd prime and if f maps the $n-1$ dimensional sphere S^{n-1} into the ℓ dimensional Euclidean space R^{ℓ} then for some $z \in f(S^{n-1})$, $f^{-1}(z)$ contains at least k mutually orthogonal points (in the obvious sense) of S^{n-1} where $\{(n - k - e(n))/(k - 1)\} > \ell$. (Received June 11, 1958.)

548-123. M. K. Fort, Jr. and G. A. Hedlund: Minimal coverings of pairs by triples.

Let F be a finite set. An F-covering of pairs by triples, or F-copt, is a set S of triples of distinct members of F such that each pair of distinct members of F is contained in at least one member of S. An F-copt S is minimal if card $S \leq$ card S' for every F-copt S'. Let F contain n elements, let S be a minimal F-copt and let $\phi(n) =$ card S. Theorem: $\phi(n)$ is the least integer which is not less than $(n/3)[n/2]$. If $n \equiv 1$ or 3 (mod 6) this is a classic result concerning Steiner Triple Systems and the coverings are exact in that each pair from F is contained in exactly one member of a minimal F-copt. In all other cases the minimal coverings are not exact. In this paper rather precise results concerning the nature of the duplications in the inexact cases are derived. (Received April 17, 1958.)

548-124. I. S. Gal: Distributions on measure spaces.

Let S be a measure space and let Y be the linear space of real or complex valued functions on S which are measurable and summable on every set of finite measure. Let X be a linear subspace consisting of essentially bounded functions x which have supports of finite measure. A uniform structure is defined for Y by a family $\mathcal{U}$ of sets $U \subseteq X$ and the corresponding norms $\|y\|_U = \sup r[|(y,u)| : u \in U]$ where $(y,u) = \int yu\,d\mu$. Assume that $\lambda U \in \mathcal{U}$ for every $\lambda \neq 0$ and $U \in \mathcal{U}$. The completion of Y with respect to $\mathcal{U}$ can be identified with a family of linear functionals on X. If X is dense in Y then the elements of Y are distributions with respect to $\mathcal{U}$ i.e. $Y \subseteq \overline{X}$. Let $A: X \to X$ be a linear operator with adjoint A^* and such that if $U \in \mathcal{U}$ then $A^*(U) \subseteq V$ for some $V \in \mathcal{U}$. Then A can be extended to a continuous linear operator $A: \overline{X} \to \overline{X}$. Let $S = E^n$ be the n-dimensional Euclidean space and let μ be the Lebesgue measure. Denote by X the linear space of real valued continuous functions with compact support which have mixed derivatives of all orders. Let $\mathcal{A}$ be the semi-group

of mixed differential operators acting on X. For several choices of $\mathfrak{A}$ the elements of $\mathfrak{A}$ can be extended to continuous operators on $\bar{X}$. Choosing $\mathfrak{A}$ properly one obtains the linear space of Schwartz distributions. (Received June 9, 1958.)

548-125. Donald Greenspan: On the numerical solution of Dirichlet problems.

A numerical method is described for the solution of n-dimensional Dirichlet problems and it is shown that the numerical solution converges to the analytic solution. The difference analogue used is $-2u(x_1,x_2,\ldots,x_n) + (\sum_1^n B_j)^{-1} \sum_1^n \{B_j [u(x_1,x_2,\ldots,x_{j-1},x_j + h_j,x_{j+1},\ldots,x_n) + u(x_1,x_2,\ldots,x_{j-1},x_j - h_j, x_{j+1},\ldots,x_n)]\} = 0$, where $B_1 = h_2^2 h_3^2 \ldots h_n^2$, $B_n = h_1^2 h_2^2 \ldots h_{n-1}^2$, $B_j = h_1^2 h_2^2 \ldots h_{j-1}^2 h_{j+1}^2 \ldots h_n^2$ $j = 2,3,\ldots,n-1$. (Received March 27, 1958.)

548-126. Edward Halpern: Hopf algebras of class 1.

Let H be an A-Hopf algebra with coproduct Δ (A a commutative ring with unit). $x \in H$ is primitive if $\Delta(x) = x \otimes 1 + 1 \otimes x$. Let $\mathfrak{P}_0$ denote the submodule of primitive elements. Filter H by the subalgebras $\mathfrak{P}^{(k)}$: $\mathfrak{P}^{(0)}$ is generated by $\mathfrak{P}_0$; for $k > 0$, $\mathfrak{P}^{(k)}$ is generated by all elements $x \in H$ such that $\Delta(x) - x \otimes 1 - 1 \otimes x \in \mathfrak{P}^{(k-1)} \otimes \mathfrak{P}^{(k-1)}$. H is of class k if k is the least integer such that $H = \mathfrak{P}^{(k)}$. THEOREM 1. Let A be a field of characteristic zero. If H is an A-Hopf algebra of class 1 and with associative Δ then there is a subset $X \subset H$ such that: (i) $\Delta(x) - x \otimes 1 - 1 \otimes x \in \mathfrak{P}_0 \otimes \mathfrak{P}_0$ (all $x \in X$), (ii) H is algebra isomorphic to the (weak) tensor product of the subalgebras generated by each x. THEOREM 2. If in addition H is finitely generated in each dimension then its dual Pontrjagin algebra is isomorphic to the free associative algebra generated by the elements of X with relations of the form: $xy = (-1)^{ij} yx + \sum a_k z_k$, where $x,y \in X$ and have degrees i,j, $a_k \in A$, and the $z_k \in X$ and are nonprimitive. In particular, the summation vanishes if (i) $x = y$ has even degree, (ii) x or y is not primitive. (Received June 16, 1958.)

548-127. A. S. Householder: Unitary triangularization of a matrix.

Lemma: For any vector $a \neq 0$, and any unit vector v, a unit vector u exists such that $(I - 2uu^*)a = \|a\|v$, where $\|a\|$ represents the Euclidean norm: $\|a\|^2 = a^*a$. The computation requires two square roots, and one reciprocation. The matrix $I - 2uu^*$ is unitary and the lemma is almost self-evident. If a is the first column of a matrix A, take $v = e_1$, the first column of the identity. The

result is the elimination of all elements below the first in the first column. Proceed with the principal submatrix remaining after suppression of the first row and column. If A has n linearly independent columns, all elements α_{ij} for $i > j$ are eliminated after at most n such steps, or n - 1 if A is square. The resulting triangle is identical with the one obtainable from application of Cholesky's square-rooting to A*A. (Received May 29, 1958.)

548-128. Frank Kozin: On the quadratic detector. I.

The purpose of this paper is to show the equivalence of the quadratic detector problem, for a certain class of stochastic processes, to the problem of taking the Wiener average of a related functional. Let $[x(t);\ 0 \leqq t \leqq T]$, $P\{x(0) = 0\} = 1$, be a separable process with stationary independent increments. Let $[y(t);\ 0 \leqq t \leqq T]$, $P\{y(0) = 0\} = 1$, be the Wiener process. For $\lambda > 0$ and with certain mild conditions on K(t), the average $E_x[\exp(-\lambda \int_0^T K(T-t)x^2(t)dt)]$ can be shown to be equal to $[K(T)/K(0)]^{1/4} E_y[\exp\{\int_0^T h(2\lambda^{1/2}(K(t))^{1/2} y(t)) - k(t)y^2(t)dt - r(T)y^2(T)\}]$, where k(t) and r(t) are given in terms of K(t) and h(u) is the Levy-Khintchine representation for processes with stationary independent increments. The proof makes use of the transformation theory of Cameron and Martin [Trans. Amer. Math. Soc. vol. 58 (1945) p. 184]. (Received July 7, 1958.)

548-129. Erwin Kreyszig: On the eigenvalues of Hermitian matrices.

The present paper is concerned with a new simple proof of the following inclusion theorem by H. Wielandt: Let A be an n-rowed Hermitian matrix. Let $x_0 \neq 0$ be an arbitrary vector with n components, $x_1 = Ax_0, m_0 = x_0^2, m_1 = x_0 x_1, m_2 = x_1^2$. Then if u is any real number and $v = (m_1 u - m_2)/(m_0 u - m_1)$ each of the two closed intervals with endpoints u and v on the projective line contains at least one eigenvalue of A. The above-mentioned proof is based upon the consideration of a certain system of inequalities. The consideration leads to a relation between that theorem and the Mohr diagram used in the theory of elasticity for representing the tensions at any point of a deformed elastic body. (Received May 2, 1958.)

548-130. K. W. Kwun: A sufficient condition that an upper semicontinuous decomposition of S^3 yield a topological 3-sphere.

If G is an upper semicontinuous decomposition of S^3 such that there are only a countable number of nondegenerate elements and their union is closed,

hen it is clear that in order that the decomposition space be a 3-sphere it is necessary and sufficient that each nondegenerate element of G has an arbitrarily close neighborhood whose boundary is a 2-sphere meeting no nondegenerate element. In this paper, we show that the above condition is sufficient regardless of the number of the nondegenerate elements. I.e., we have: Let G be an upper semicontinuous decomposition of S such that the closure F of the union of the nondegenerate elements of G corresponds to a totally disconnected set in the decomposition space. If each component of F has an arbitrarily close neighborhood whose boundary is a 2- sphere lying in the complement of F, then the decomposition space is a 3-sphere. I understand that Professor O. G. Harrold, Jr. has obtained the identical result. (See Abst. 547-24. Notices Amer. Math. Soc., August, 1958). (Received July 7, 1958.)

548-131. Joseph Lehner: On modular forms of dimension -2.

Modular forms of dimension -2 can be expanded in Poincaré series which do not converge absolutely. In 1927 Hecke introduced a convergence factor which has since been used successfully by Petersson for the investigation of these forms. In the present paper the author offers a new approach to the problem by proving that the Poincaré series converges without benefit of the convergence factor if summed in the proper order. The convergence of the series is uniform and it is shown directly that the sum of the series is a modular form. The series is then rearranged in a Fourier series and one obtains the usual Petersson - Rademacher expansions of the Fourier coefficients. The method depends critically on the estimation of certain Kloosterman exponential sums. The method still works for forms of dimension less than -3/2, provided the appropriate exponential sums can be estimated as $O(k^{1/2+\epsilon})$, where k is the modulus of the sum. As an example, the form $J'(\tau)$, of dimension -2, has the following convergent Poincaré series: $-12^3 J(\tau)/\pi i = \exp\{-2\pi i\tau\} + \lim_{K\to\infty} \sum_{k=-K}^{K} \cdot \sum_{m=-\infty}^{\infty} (k\tau - m)^{-2} \exp\{-2\pi i(m'\tau - k')/(k\tau - m)\}$, where $\operatorname{Im}\tau > 0$, $k \neq 0$, $(m,k) = 1$, and $kk' - mm' = 1$. (Received June 18, 1958.)

548-132. G. O. Losey: On the decompositions of a group algebra into a sum of a field and a nil ideal.

Let G be a group, K a field of characteristic p and Γ the group algebra of G over K. In a recent article, Kaplansky has asked the following question: If

Γ can be decomposed into a direct sum of K-modules, $\Gamma = K' \oplus \Lambda$, where $K' \cong K$ and Λ is a nil ideal of Γ, is G a locally finite p-group? We prove the following. Theorem. G is a locally finite p-group if and only if $\Gamma = K' \oplus \Lambda$, where $K' \cong K$ and Λ is a nil ideal of Γ in which every finitely generated subalgebra has a bounded index of nilpotency. To prove this it was necessary to prove: Theorem. G is a finite p-group if and only if $\Gamma = K' \oplus \Lambda$, where $K' \cong K$ and Λ is a nilpotent ideal of Γ. (Received July 7, 1958.)

548-133. J. E. McLaughlin: Prime power representations of finite simple Lie groups.

Let G be one of the simple groups defined by Chevalley (Sur certains groupes simples, Tôhoku Math. J. vol. 7 (1955)) over a finite field of characteristic p. Then if P, of order d, is a p-Sylow subgroup of G and m is the index of the normalizer of P in G, G has an irreducible representation of degree d in any field whose characteristic does not divide m. This extends a theorem of Steinberg (Prime power representations of finite linear groups, Canad. J. Math. vol. 8 (1956)) to groups of type G_2, F_4, E_6, E_7, E_8. The method of proof here is uniform for all of the groups. (Received June 2, 1958.)

548-134. Morris Marden: On the zeros of infrapolynomials for partly arbitrary point sets.

Let the point set S consist of a given compact point set S_0 and p additional points situated at arbitrary positions in the complex plane. Let the nth degree polynomial $f(z) = z^n + a_1 z^{n-1} + \ldots + a_n$ be an infrapolynomial upon S in the sense that f(z) has no underpolynomial $g(z) = z^n + \ldots$ upon S. It is proved in this note that at most p zeros of f(z) may lie outside the region comprised of all points from which S_0 subtends an angle of at least $\pi/(p+1)$. The proof is based upon theorems of Fekete [Proc. Nat. Acad. Sci. vol. 37 (1951) pp. 95-103] and Marden [Trans. Amer. Math. Soc. vol. 45 (1939) pp. 355-368]. When $p = 0$, this theorem reduces to Féjer's Theorem [Math. Ann. vol. 85 (1922) pp. 41-48] that all the zeros of f(z) lie in the convex hull of S. (Received July 9, 1958.)

548-135. Z. A. Melzak: Entire operators and functional equations.

Let $J = [0,\infty)$ and let C, L' and L^∞ denote respectively the sets of continuous, integrable, and essentially bounded functions on J. Put $F = C \cap L' \cap L^\infty$. For $n \geq 1$ let K_n be a mapping such that (1) K_n: $F \times F \times \ldots \times F$(n times) $\to C$,

(2) for each x, $K_n(f_1,\dots,f_n)$ is a symmetric n-linear operator, (3) $\|K_n(f_1,\dots,f_n)\|_1 \leq C_n\prod_1^n\|f_i\|_1$, (4) $\|K_1(f)\|_\infty \leq D_1(\|f\|_1 + \|f\|_\infty)$ and (5) for $n \geq 2$, $\|K_n(f_1,\dots,f_n)\|_\infty \leq D_n\prod_1^n\|f_i\|_1\sum_1^m\|f_i\|_\infty/\|f_i\|_1$. C_n and D_n are nonnegative constants. Let $K_n(f) = K_n(f,\dots,f)$. If the two series $\sum_1^\infty C_n z^n$ and $\sum_1^\infty D_n z^n$ represent entire functions then $K(f) = \sum_1^\infty K_n(f)$ is called an entire operator. Existence, uniqueness and infinite differentiability theorems are proved for the initial value problem $\partial f(x,t)/\partial t = K[f(x,t)]$ where $x,t = 0$, $f(x,0)$ is known, and K is an entire operator. Equations of that type occur in transport theory; K_n for $n \geq 2$ is then the contribution toward the rate of reshuffling, due to n-tuple collisions, while K_1 is a break-up term. (Received July 7, 1958.)

548-136. L. J. Mordell: On the inequality $\sum_{r=1}^n (x_r/(x_{r+1}+x_{r+2})) \geq n/2$ and some others.

This inequality, where the x are non-negative real numbers and $x_{n+1} = x_1$, $x_{n+2} = x_2$, was originally proposed by H. S. Shapiro in 1954 in the American Mathematical Monthly, and has attracted considerable attention. It is known to be true for $n = 3,4,5$, and false for $n = 20$. I prove the inequality very simply for $n = 3,4,5,6$ and conjecture that it is false for $n \geq 7$. The proof follows at once from the Theorem. Let $x_1,x_2,\dots,x_n$ be non-negative real numbers and let $x_{n+1} = x_1, x_{n+2} = x_2$. Then $(\sum_{r=1}^n x_r)^2 \geq j(\sum_{r=1}^n x_r(x_{r+1} + x_{r+2}))$, where $j = \min(n/2,3)$ is the best possible constant. This theorem is one of a chain. Thus we have the Theorem. If also $x_{n+3} = x_3$, then $(\sum_{r=1}^n x_r)^2 \geq k(\sum_{r=1}^n x_r(x_{r+1} + x_{r+2} + x_{r+3}))$, where $k = \min(n/3, 8/3)$ is the best possible constant. These results lead to others. Thus there is the Theorem. If the k are non-negative real numbers, $\sum_{r=1}^n (x_r^2 - 2x_r x_{r+1}) \geq l(\sum_{r=1}^n x_r)^2$, where $l = -\max(1/n,1/6)$ is the best possible constant. An account of all this is appearing in the Artin 60th birthday number of the Hamburger Abhandlung, 1958. (Received July 7, 1958.)

548-137. J. R. Munkres: Differentiable isotopies on the two-sphere.

Given two C^1 nondegenerate homeomorphisms of one differentiable manifold onto another, a C^1 isotopy is a homotopy f_t between them which is, for each t, a C^1 nondegenerate homeomorphism onto, and is differentiable in t. The following theorem is proved: Given an orientation-preserving C^1 homeomorphism f of S^2 onto itself, there is a C^1 isotopy f_t between f and the identity. The corresponding theorem for S^6 is not true (J.W.Milnor, Ann. of Math. vol. 64 (1956)

p. 404). If the condition C^1 is removed from hypothesis and conclusion, the theorem becomes a known one, the techniques involved in whose proof are quite different from those of the present one. (Received July 10, 1958.)

548-138. J. W. Neuberger: Concerning boundary value problems.

This paper follows the work on integral equations of H. S. Wall, J. S. MacNerney and the present author. Suppose [a,b] is a number interval and S an abelian, normed, complete space. A necessary and sufficient condition that for each point C in S and each continuous function g from [a,b] into S, there be a unique function Y from [a,b] into S such that (A) $Y(t) = Y(u) + g(t) - g(u) + \int_u^t dF \cdot Y$ and $\int_a^b dH \cdot Y = C$ for each of t and u in [a,b] (each of H and F a function of bounded variation from [a,b] into the space B of all bounded linear transformations from S into S, and F continuous) is that for some u in [a,b], $\int_a^b dH(s) \cdot M(s,u)$ take S onto S reversibly, where M satisfies $M(t,u) = I + \int_u^t dF(s) \cdot M(s,u)$, for each of t and u in [a,b], with I the identity in B. A necessary and sufficient condition is given that there be a unique solution to a certain boundary value problem which includes (A) under certain conditions not requiring linearity of F(t) and H(t) for t in [a,b]. An application to Sturm-Liouville boundary value problems is given. (Received June 30, 1958.)

548-139. E. C. Posner: An alternate characterization of the lower nil radical.

It is known or easily provable that if a ring has no nilpotent ideals, it has no nil ideals of bounded index. (In fact every nil left ideal of bounded index contains a nilpotent left ideal.) Now imitate the definition of the lower nil radical, except replace "nilpotent ideal" by "nil ideal of bounded index". Then this object contains, and is contained in, the lower nil radical, by the result of the first sentence of this abstract. Throughout the preceding, we can replace "ring" by "algebra" and "nil...of bounded index" by "nil PI...". (Received July 10, 1958.)

548-140. Walter Prenowitz: A theory of incidence geometries.

Theories of incidence geometry have been given by Gorn (Bull. Amer. Math. Soc. vol. 46 (1940) pp. 158-167) using generalized lattices, and by Wyler (Duke Math. J. vol. 20 (1953) pp. 601-610) using point and plane as undefined. In this paper an incidence geometry is a system (S,T), where S and T are sets whose respective elements are called points and lines, which satisfies the following

three postulates: (1) A line is a set of points containing at least two points. (2) Two distinct points belong to a unique line. Definition. The join ab of points a,b ($a \neq b$) is the unique line containing a and b; the join aa of a and a consists of a. If A,B are nonvoid sets of points their join AB is the set union $\bigcup_{a\in A, b\in B}(ab)$. (3) (ab)((ac)(ad)) = ((ab)(ac))(ad), for any points a,b,c,d. Examples of incidence geometries are projective, descriptive (linear ordered) and affine geometries. A, a set of points, is linear if $x,y \in A$ implies $xy \subset A$. The theory involves no dimensional restriction or existence assumption and is reasonably broad, but relatively pathological geometries which are covered by the theories of Gorn and Wyler are excluded by the Theorem: If A,B are linear and intersect then AB is linear. (Received July 10, 1958.)

548-141. WITHDRAWN.

548-142. Irving Reiner: On the class number of representations of an order.

Let $\mathfrak{o}$ be a Dedekind ring with class number h and quotient field K. Let A be a direct sum of full matrix algebras over K, and let G be an $\mathfrak{o}$-order in A. Form the nonzero ideal $\mathfrak{f}$ in $\mathfrak{o}$, defined as the intersection of the annihilators of the one-dimensional cohomology groups H(G,T) as T ranges over all two-sided G-modules. Now let L' be a left A-module having k distinct irreducible constituents, and let S be the set of all left G-modules L which are finitely-generated torsion-free $\mathfrak{o}$-modules such that KL = L'. Partition S into s classes under G-isomorphism, and into r genera. Extending a result of Maranda's (Canad. J. Math. vol. 7 (1955) pp. 516-526), it is shown here that $s \leqq h^k r$, with equality holding under the following hypothesis: For each $a \in \mathfrak{o}$ with $(a) + \mathfrak{f} = \mathfrak{o}$ there exists a unit $b \in \mathfrak{o}$ with $b \equiv a \pmod{\mathfrak{f}^{k-1}}$. Explicit formulas for r and s are obtained for the case k = 2 to show that this hypothesis is not superfluous. (Received June 4, 1958.)

548-143. R. W. Richardson, Jr.: Actions of the groups SO(3) and Sp(1) on the five-sphere.

A study of the actions of SO(3) and Sp(1) on the five sphere, S^5, is made. It is shown that, to within equivalence, there is exactly one effective action of Sp(1) on S^5. This action is linear. There are either two or three equivalence classes of actions of SO(3) on S^5 such that there exists a three-dimensional

orbit. Two of these actions are linear. In the possible nonlinear action of SO(3) on S^5 a space M is constructed on which SO(3) acts with a three-dimensional orbit. M is of the same homotopy type as S^5 and has the same local homology and homotopy groups. The problem of whether or not there exists an action of SO(3) on S^5 not equivalent to a linear action is reduced to the problem of whether or not M is homeomorphic to S^5. This problem is not solved. The method is essentially that of the preceding paper. Using an example of Bing (Ann. of Math. vol. 65 (1957) pp. 484-500) an example is constructed which indicates the difficulty of any such classification of the actions of SO(3) on S^5 such that the highest dimension of any orbit is two. The work for these two papers has been sponsored in part by A. F. 49 (638)-104. (Received July 11, 1958.)

548-144. J. R. Stallings: Unknotted curves on 2-manifolds in 3-manifolds.

Let T be a collection of finitely many, disjoint, semilinearly embedded, orientable 2-manifolds T_i, in the orientable 3-manifold without boundary M. Genus $T = \sum$ Genus T_i. A simple closed curve $C \subset M$ is M-unknotted if it bounds a polyhedral disk in M; it is (M - T)-unknotted if this disk can be chosen so that its interior is disjoint from T. In this situation, a result of Fox [Ann. of Math. vol. 49 (1948) pp. 462] can be generalized as follows: Theorem 1: If each (M - T)-unknotted curve $C \subset T$ is contractible on T, then each of the injections $\pi_1(T_i) \longrightarrow \pi_1(M)$ has kernel 1. Theorem 2: If $\pi_1(M)$ contains no subgroup isomorphic to the fundamental group of any nonspherical orientable 2-manifold, and k = Genus T, then on T there are k nonintersecting simple closed curves $C_1,\dots,C_k$, which are M-unknotted and linearly independent with respect to homology on T. The proof uses the theorems of Papakyriakopoulos [Proc. London Math. Soc. vol. 7 (1957) p. 281; Ann. of Math. vol. 66 (1957) p. 1]. (Received July 8, 1958.)

548-145. J. R. Stallings: Commutators in free products with amalgamation.

Questions about commutators in groups can be translated into questions on the topology of orientable 2-manifolds. Using this technique, it can be shown: Theorem 1: Let $G *_A H$ be the free product of G and H with amalgamated subgroup A. Suppose $g \in G$, $g = [u_1,v_1]\cdot\dots\cdot[u_n,v_n]$, where $u_i,v_i \in G *_A H$. And suppose that g cannot be written as the product of k commutators of elements of G times the product of m conjugates of elements of A by elements of G, where $k < n$ and $m \leqq 2(n - k)$. Then $u_i \in G, v_i \in G$. Theorem 2: Let $H = G_1 * \dots * G_n$,

$h = g_1 \cdot \ldots \cdot g_n$, where $1 \neq g_i \in G_i$. And suppose h is the product of k commutators of elements of H. Then $k \geqq n$. If k is the minimal integer for such an expression of h, then $k = k_1 + \ldots + k_n$, such that g_i is the product of k_i commutators. Theorem 1 generalizes a lemma of Kurosh [Mat. Sb. N. S. vol. 2 (1937) p. 995], and the first part of Theorem 2 is a theorem of Griffiths [Proc. Cambridge Philos. Soc. vol. 51 (1955) p. 245]. (Received July 8, 1958.)

548-146. Paul Stein: On a class of quadratic transformations.

A class of quadratic transformations in N dimensions (with an interpretation of possible interest in mathematical biology) has been studied. The transformations are of the form: $x'_{ij} = \sum_{ij}^{k\ell mn} x_{km} x_{\ell n}$ (1), where the sum is over all k,ℓ,m,n from 1 to N. The coefficients $\gamma_j^{k\ell}$ are positive for $k \leqq j \leqq \ell$, zero otherwise; their sum on j is 1; the moment of the distribution of the γ_j is $(k+\ell)/2$. The x_{ij} are taken non-negative, symmetric in i,j, with sum = 1. Under these conditions, the following results are obtained: for any initial set $x_{ij}^{(o)}$, the iterates of the transformations converge to a unique fixed-point which may be explicitly computed from the initial set $x^{(o)}$. This fixed-point has the unusual property that all the x_{ij} are zero except for four terms: x_{kk}, $x_{k+1,k} = x_{k,k+1}$, $x_{k+1,k+1}$, whose values depend on a single real parameter which is an explicit function of the initial set. The value of k is also explicitly given. The transformation has been generalized by replacing x_{ij} by a quantity $x_{ijk\ldots\ell}$ with M indices, completely symmetric in all of them. Analogous results then hold, the only nonvanishing x's at the uniquely determined fixed-point lying on and immediately adjacent to a (generalized) diagonal. (Received June 2, 1958.)

548-147. C. E. Stewart: On the convergence of sequences of singular integrals of Caughy type. Part II, polygonal approximations.

A polygon p is a polygonal approximation to a function f on [a,b], if p is f at the corners of p, i.e., $p(x_k) = f(x_k)$ where p is linear on the subintervals $[x_k, x_{k-1}]$. The maximum of the lengths of the intervals over which p is linear is defined as the norm of the spacing of p. THEOREM I: If $\{p_n\}_{n=1}^{\infty}$ is a sequence of polygonal approximations to a continuously differentiable function f on [a,b], such that the norm of the spacing of p_n tends to zero as $n \to \infty$, then $\{p_n\}_{n=1}^{\infty}$ is S*-convergent on [a,b]. (See Part I.) THEOREM II: If $\{p_n\}_{n=1}^{\infty}$ is a sequence of polygonal approximations to a continuously differentiable function f on [a,b], such that the norm of the spacing of p_n tends to zero as $n \to \infty$, then for each y in (a,b) the

Cauchy principal value of $p_n^{(1)}(y) = \int_a^b (x - y)^{-1}((b - x)(x - a))^{-1/2} p_n(x)\,dx$ exists, and the sequence $\{p_n^{(1)}\}_{n=1}^{\infty}$ is uniformly convergent on (a,b) to the function $f^{(1)}$, where for y in (a,b), $f^{(1)}(y) = \int_a^b (x - y)^{-1}((b - x)(x - a))^{-1/2} f(x)dx$. (Received June 19, 1958.)

548-148. C. E. Stewart: On the existence of singular integrals of Cauchy type.

We consider a Cauchy principal integral of the form (i) $\int_a^b (x - y)^{-1} f(x)\,dx$ where y is a point of (a,b). The familiar Plemelj-Privalov Theorem (Muskhelishvili, N.I., Singular integral equations, Translation edited by J. R. M. Radok; P. Noordhoff, N. V. Holland, 1953) asserts that the principal value in (i) exists for each f in any Hölder class $\mu, 0 < \mu \leqq 1$. Such functions are, of course, continuous. Examples of continuous functions which do not belong to any Hölder class are given. For these particular functions, it is shown that (i) does not exist at certain points y in (a,b). In one case, (i) does not exist on a dense subset of (a,b). (Received June 19, 1958.)

548-149. P. M. Swingle: Lebesgue nonmeasurability and connected locally connected algebras.

Let the imbedding space be the euclidean E_m, $m \geqq 2$. Then using the hypothesis of the continuum and Zermelo's well ordering theorem one can show: There exists densely in any connected domain D of E_m a connected set V of Vitali (Notices, Amer. Math. Soc. April, 1958, Abstract 542-66) which contains densely a connected locally connected semigroup C; or C algebraically can be a ring or a field inheriting its · and + from the ordinary coordinate multiplication and addition of E_m. (Received July 10, 1958.)

548-150. L. B. Treybig: Concerning local separability in locally peripherally separable spaces.

In his dissertation, (University of Texas, June, 1958), the author gives an example of a connected, locally peripherally separable, metric space which is not separable, and which has the property that the set of all points at which it is not locally separable is separable. In the present paper the author shows that if Σ is any connected, locally peripherally separable, metric space which is not separable, then the set of all points at which Σ is not locally separable is uncountable. (Received June 16, 1958.)

548-151. Mary Weiss: On the law of the iterated logarithm.

The Law of the Iterated Logarithm asserts that if $X_1, X_2, \ldots, X_n, \ldots$ are independent random variables with first moments 0, and second moments $b_1, b_2, \ldots, b_n, \ldots$ and if (*) $\max|X_n| = o\{B_n(\log \log B_n)^{-1/2}\}$ where $B_n^2 = \sum_1^n b_i$, then, with probability 1, (**) $\lim_n \sup (X_1 + X_2 + \ldots + X_n)(2B_n^2 \log \log B_n)^{-1/2} = 1$. It can be shown that there exists a sequence of independent random variables satisfying (***) $\max|X_n| \cong K\{B_n(\log \log B_n)^{-1/2}\}$ instead of (**), for which $\lim_n \sup(X_1 + X_2 \ldots X_n)(2B_n^2 \log \log B_n)^{-1/2} > 1$, with probability 1. (That we can have lim sup < 1 is an old result of Marcinkiewicz). (Received June 16, 1958.)

548-152. Mary Weiss: On a problem of Littlewood.

Settling an old conjecture of Steinhaus, Helson proved recently (Proc. Nat. Acad. Sci. U. S. A. vol. 40 (1954) pp. 205-206) that if the partial sums, s_n, of a trigonometric series $\sum a_n \cos nx + b_n \sin nx$ satisfy (*) $\int_0^{2\pi} |s_n(x)|dx = O(1)$ then a_n and b_n tend to zero. The question as to whether (*) implies that the series is a Fourier Series has been raised. It is shown that this is not necessarily the case and the counterexample is provided by the Riesz product $\prod(1 + k^{-1/2}\cos 4^k x)$. (Received June 16, 1958.)

548-153. Mary Weiss: On the law of the iterated logarithm for uniformly bounded orthonormal systems.

Given any system of functions $\phi_1, \phi_2, \ldots, \phi_n, \ldots$ real-valued, uniformly bounded, and orthonormal on the interval (0,1), there exists a subsequence $\phi_{n_1}, \phi_{n_2}, \ldots$ and a function $f(x) \geq 0$ with the following properties. If $c_1, c_2, \ldots, c_n, \ldots$ is a sequence of real numbers, $C_n^2 = (c_1^2 + \ldots c_n^2) \to \infty$, $c_n = o\{C_n(\log \log C_n)^{-1/2}\}$ then $S_k = c_1\phi_{n_1} + \ldots + c_k\phi_{n_k}$ satisfies a.e. the relation $\lim \sup\{S_k(x)(2C_k^2 \cdot \log \log C_k)^{-1/2}\} = f(x)$. The function $f^2(x)$ is the weak limit of $\{\phi_{n_k}^2(x)\}$. (Received June 16, 1958.)

548-154. A. B. Willcox: On homogeneous algebras of functions.

Generalizations are obtained for some of G. E. Silov's results on homogeneous algebras of functions over a compact abelian group (Amer. Math. Soc. Translations no. 92, 1953). For G a locally compact, σ-compact abelian group, K a primary B-algebra with 1 and maximal ideal Q, and ω a homomorphism of $\hat{G}$ into K with $\omega(\hat{G}) \neq 1 \pmod{Q}$ a B-algebra $K_\omega(G)$ is defined, the definition reducing to Silov's in case G is compact. $K_\omega(G)$ can be represented as a B-algebra

of K-valued continuous functions vanishing at ∞ on G with the sup norm. $TK_\omega(G)$ is the closure of the set of elements of $K_\omega(G)$ with compact support. If $TK_\omega(G)$ separates points of G and ω is continuous then G is the structure space of $TK_\omega(G)$. For certain groups (e.g., connected) necessary and sufficient conditions for a regular $TK_\omega(G)$ to be semi-simple and hence of type C are obtained. If the regular commutative B-algebra R is homogeneous (Silov, 5.1), Tauberian, and of type C over a connected l.c. abelian Group G then R is closed under multiplication by $\hat{G}$ and is locally isomorphic to an algebra $TK_\omega(G)$. If, in addition, R has a bounded set of units (compact support) modulo the sets in a σ-covering of G then $R = K_\omega(G)$. For certain special cases similar results are possible for the C-completion of a nontype C algebra of the above kind. Analogs of all of Silov's examples of homogeneous algebras are discussed. (Received June 24, 1958.)

548-155. R. G. Ayoub: Ideals and the Dedekind zeta function.

Let K be an algebraic extension of the rationals of degree k, and let $\zeta(S,K)$ denote the Zeta function for the field K; i.e., $\zeta(S,K) = \sum 1/N(\alpha)^S = \sum F(n)/n^S$; F(n) is the number of ideals with norm equal to the rational integer n. H(x) is the number of ideals with norm less than or equal to x; i.e. $H(x) = \sum_{n \leq x} F(n)$. If α is the residue of $\zeta(S,K)$ at $s = 1$, then $H(x) = \alpha x + \Delta(x)$. The relation between $\Delta(x)$ and the Lindelof hypothesis for $\zeta(S,K)$ isexamined and a necessary and sufficient condition for the truth of the hypothesis is established in the case when the field is abelian and the degree is sufficiently high. (Received July 14, 1958.)

548-156. W. T. Guy, Jr.: Operational solution of certain integral equations.

An operational technique of M. Parodi used in solving certain Fredholm type integral equations is extended. This technique is used in solving a certain heat equation problem. (Received July 14, 1958.)

548-157. D. W. Miller: An imbedding theorem for a class of nets. Preliminary report.

It is well known that every net of degree n, order n, with $n \geq 3$, can be imbedded in a unique affine plane of order n. R. H. Bruck has obtained a necessary and sufficient condition for the imbeddability of a net of degree n - 1, order n in an affine plane of order n. Let $\mathcal{N}$ be a net of order $n > 9$, degree n - 2. If

P,Q are distinct collinear points of $\mathcal{H}$ there are exactly six points which are collinear with neither P nor Q in $\mathcal{H}$. The following condition is shown to be both necessary and sufficient for $\mathcal{H}$ to be imbeddable in an affine plane of order n: For each pair P,Q of distinct collinear points of $\mathcal{H}$, the points which are collinear with neither P nor Q in $\mathcal{H}$ form two "triangles", i.e. for a suitable labeling of these points, $R_1 \wedge R_2, R_2 \wedge R_3, R_3 \wedge R_1$ and $R_4 \wedge R_5, R_5 \wedge R_6, R_6 \wedge R_4$ (where $R_i \wedge R_j$ denotes that R_i and R_j are collinear in $\mathcal{H}$). (Received July 14, 1958.)

548-158. W. J. Riordan: On the interpolation of operations.

For any function, f, let $d_f(y) =$ measure of $\{x: |f(x)| > y\}$. A linear transformation T is said to be of weak type (a,b) if $d_{Tf}(y) \leq [(k/y)\|f\|_a]^b$. T is said to be of type (a,b) if $\|Tf\|_b \leq K\|f\|_a$. A theorem of Marcinkiewicz states that if T is of weak types (a_1,b_1) and (a_2,b_2) then it is of type (a,b) for all (a,b) satisfying $1/b = t/b_1 + (1-t)/b_2$; $1/a = t/a_1 + (1-t)/a_2$, $0 < t < 1$. This theorem on L_p spaces is generalized to L_ϕ spaces, where ϕ is any convex increasing function. Suppose ψ and ϕ are two such functions, such that $\psi(x)$ is "between" x^{b_1} and x^{b_2}, and $\phi(x)$ is "between" x^{a_1} and x^{a_2}, and they also satisfy a certain relation involving a_1, a_2, b_1, b_2. Then if T is of weak types (a_1,b_1) and (a_2,b_2) it is of type (ϕ,ψ), that is $\|Tf\|_\psi \leq K\|f\|_\phi$ for all $f \in L_\phi$. There are applications in fractional integration and Fourier transforms. (Received July 14, 1958.)

548-159. Daniel Waterman: On the convergence of the Laplace-Stieltjes transform.

A theorem of Hardy and Littlewood asserts that if C is a curve in $|z| < 1$, terminating at $z = 1$, and with monotone ordinate, then the existence of $\lim \sum a_n z^n$ as $z \to 1$ along C implies, under the condition $a_n = o(1/n)$, that $\sum a_n$ converges, no matter how steep C may be. A generalization of this result to the Laplace-Stieltjes transform may be proven. If $f(s) = \int_0^\infty e^{-st} d\phi(t)$ and C is similarly defined in $R(s) > 0$, then with suitable restrictions on ϕ, $\int_0^\infty d\phi(t)$ exists if $\lim f(s)$ exists as $s \to 0$ along C. The restriction made is that $\sup|\phi(x+h) - \phi(x)|$ over the interval $0 < h \leq 1$, $= o(1/x)$. The proof uses equiconvergent trigonometrical series. The condition most clearly analagous to that for power series is more restrictive; it is that the variation of ϕ in $[x, x+1]$ be $o(1/x)$. Under this condition the proof of the result parallels that of the Hardy-Littlewood theorem as given by Landau. (Received July 14, 1958.)

548-160. A. J. Penico: Guided monochromatic waves in an anisotropic medium.

Under certain conditions, the behavior of a monochromatic electromagnetic disturbance in an anisotropic medium is determined from the solution of the equation (A) curl curl $\mathcal{E}$ - S$\mathcal{E}$ = 0 , where $\mathcal{E}$ is the electric field vector and S is a linear transformation, independent of time and space coordinates, acting on $\mathcal{E}$. Relative to an appropriate coordinate system, S has a matrix representation with components $s_{11}, s_{22}, s_{33}, s_{12} = -s_{21}, s_{13} = s_{31} = s_{23} = s_{32} = 0$. The case is considered here of a wave propagating in the z-direction through a waveguide having a uniform cross-section and infinitely conducting walls. The direction of the axis of the waveguide is taken as the z-direction. An arbitrary orthogonal coordinate system is admitted in the plane perpendicular to the z-axis. A fourth-order partial differential equation for E_z, the z-component of the electric field, is then derived from (A). The aforementioned fourth-order equation for E_z is not, in general, amenable to solution by separation of variables. However, approximate solutions to this equation are obtained using variational or integral equation techniques, according to the specific cross-section under consideration. Specific cases considered are waveguides of rectangular, circular and (circular) coaxial cross-section. (Received July 16, 1958.)

548-161. Casper Goffman: Hausdorff measure and Lebesgue area.

It is known that, for a continuous nonparametric surface, the Lebesgue area equals the Hausdorff two dimensional measure of the graph of the surface. The author has extended the definition of Lebesgue area to all surfaces given by measurable functions. It is shown that, for a class of surfaces called linearly continuous, this area is equal to the Hausdorff two dimensional measure of a certain "essential" part of the graph of the surface. (Received July 17, 1958.)

548-162. P. C. Rosenbloom: Probability, entire functions, and statistical mechanics.

We show how Wiman's theorem on the relation between the maximum modulus and the maximum term in a power series is a simple consequence of Chebyshev's inequality in the theory of probability. The same method can be applied to functions of several variables, and to problems in statistical mechanics in Khinchin's formulation. (Received August 1, 1958.)

548-163. J. G. Horne: Rings with effective O-ideal structure.

Loosely, a ring R has effective O-ideal structure provided its maximal O-ideal space is homeomorphic with its structure space. In the present instance the notion of O-ideal is based on a generalization of Kohls' definition of relative identity (Fund. Math. vol. 45 (1957) pp. 17-27) which is of interest in its own right: e is a relative identity for f if for every g in the principal ideal [f], eg = ge = g. Using this definition, Lemma 4.4 and Theorems 4.7 and 4.9 of the above paper are valid for arbitrary rings. The maximal O-ideal space is the set M(R) with the topology Γ which is a modification of the dual Stone topology. The structure space of R is the set S(R) of maximal regular ideals of R with the Stone topology Σ. A ring is a local identity ring (l.i. ring) if $S \in S(R)$ implies there exists $f \in J$ which has a relative identity. R is s.s.s. if $\bigcap S(R)$ is zero. R is a G.S. ring if R is a l.i. ring and S(R) is T_2(see Wilcox, Pacific J. Math. vol. 6 (1956) pp. 177-192). If $S \in S(R)$, L(S) denotes the maximal O-ideal (if any) contained in S. THEOREM. If R is s.s.s. these are equivalent: (1) R is a G.S. ring; (2) $L:S(R)_{\Sigma} \rightarrow M(R)_{\Gamma}$ is a homeomorphism onto. Other equivalences are obtained. COROLLARY. If R is a s.s.s. ring with identity then S(R) is T_2 if and only if R has effective O-ideal structure. (Received August 4, 1958.)

548-164. Abraham Charnes and W. W. Cooper: Extremal principles for simulating traffic flow in a network.

New poly-extremal (game-theoretic) and uni-extremal principles are developed for simulating the flow of traffic in street (or highway) networks. (A. Charnes and W. W. Cooper, Proc. Nat. Acad. Sci. U. S. A., February, 1958). These conform to Wardrop's requirements (J. G. Wardrop, Journal Institution of Civil Engineers, London, 1952) and eliminate Prager's difficulties (W. Prager Proc. Midwest Research Inst., Kansas City, 1954). The new formulation and multi-copy network idea provides evaluation and computational efficiencies of mathematical (even linear) programming as well as new connections and methods for classical (e.g. plastic limit analysis) problems and for management theoretic (e.g. financial planning) problems. (Received August 6, 1958.)

548-165. F. B. Jones: Another cutpoint theorem for plane continua.

If the compact plane continuum M does not separate the plane and no point cuts between the points a and b of M, then both a and b belong to the closure of some component of the interior of M. (Received August 6, 1958.)

548-166. Abraham Charnes and C. E. Lemke: The continuous-limit method I. Minimization of convex functionals over convex polyhedra.

In Charnes and Lemke (NRLQ June 1954, Minimization of nonlinear separable convex functionals) and Charnes and Cooper (Econometrica, January 1956, Nonlinear power of adjacent extreme point methods; NRLQ September 1958, Convex programming over incidence matrices) particular stress is laid on approximation of convex programming problems by linear programming problems and exploitation of information in terms of the vector bases. This paper presents one phase of refinement to the "continuous-limit"; a convergent method which proceeds on bases but introduces a nonlinear step at each iteration. It differs from methods suggested by Beale, Frank and Wolfe and others (in context of more special problems); a noteworthy advantage is minimization of the number of basis changes. A corollary is another method for linear programming. A second (dual) phase, refinement of polyhedra to general convex sets will be dealt with elsewhere. (Received August 11, 1958.)

548-167. G. O. Sabidussi: On the set of graphs with a finite number of given blocks.

For a given cardinal λ let X_λ be the set of all nonisomorphic graphs with less than λ edges. For $x,y \in X_\lambda$ let $x \leqq y$ iff (i) there is a homeomorphism of x into y, and if (ii) $\alpha_0(x) \leqq \alpha_0(y)$, where $\alpha_0(x)$, $\alpha_0(y)$ denote the cardinals of the sets of vertices of x and y, respectively. ((ii) is a technicality which does not in any way restrict (i), but permits the statement that $\leqq$ is a partial order). It has recently been proved that the set T of all nonisomorphic finite trees is of finite width with respect to $\leqq$. We generalize this result as follows. For a given finite subset Y of mutually unrelated cyclically connected graphs in X_λ, let $X_\lambda(Y)$ be the set of all graphs $x \in X_\lambda$ with the property that (i) x has only finitely many blocks, and (ii) if z is a block of x, then either x consists of a single edge or $z \cong y$ for some $y \in Y$. Then $X_\lambda(Y)$ has finite width with respect to $\leqq$. (Received August 27, 1958.)

548-168. L. N. Mann: On a theorem of Floyd. Preliminary report.

Floyd (Ann. of Math., vol. 66) proves the following theorem: If a torus group G operates on a compact manifold X with the reals mod 1 homology of S^n, then the fixed point set F of G has the reals mod 1 homology groups of an r-sphere for some $-1 \leqq r \leqq n$. In this paper the author replaces the condition

that X be a manifold in the above theorem by less restrictive homological conditions on X. A homological manifold with respect to R (reals mod 1) is defined as a compact, connected Hausdorff space of finite homological dimension with respect to R which is HLC over R and has property P(R) (Yang, Trans. Amer. Math. Soc. vol. 87 (1958)). The above theorem is proven for the case where X is assumed to be a homological manifold with respect to R instead of an actual manifold. The basic proposition of the paper is the following: If a finite solvable group operates on a compact Hausdorff space of finite homological dimension with respect to R which is HLC over R, then the orbit space is also HLC over R. With this proposition the procedures of Floyd are used with slight modifications to obtain the desired result. (Received August 6, 1958.)

548-169. Abraham Charnes and W. W. Cooper: The strong Minkowski-Farkas-Weyl theorem for vector spaces over ordered fields.

Subject theorem is: In a finite-dimensional space, the bounded intersection of finitely many half-spaces is the convex hull of finitely many extreme points. Usual M-F-W Theorem, an immediate corollary, has been proved many times with use either of topologic notions and real fields or elaborate indirect algebraic argument. A brief constructive algebraic proof of the stronger theorem is presented using nothing more sophisticated than linear independence. (Received August 6, 1958.)

548-170. A. I. Weinzweig: The fundamental group of the union of spaces.

Let $X = A_1 \cup \ldots \cup A_n$ where, for simplicity, X is a finite simplicial complex and the A_i, open subcomplexes of X. Let N be a geometrical realization of the classical nerve of the covering $\{A_i\}$ and f: $X \to N$ the usual map. Denote by σ_s the unique open simplex of N containing $s \in N$, $G \equiv \{\langle x,s\rangle \in X \times N \mid f(x) \in \sigma_s\}$. An equivalence relation R is defined in G by $\langle x,s\rangle \equiv \langle y,t\rangle$ mod R iff $s = t$ and x,y are in the same arc component of $f^{-1}(\sigma_s)$. Set $Y = G/R$ with the quotient topology. Y is a regular cell complex. A map g: $X \to Y$ is defined by $g(x) = \langle x, f(x)\rangle$ mod R. Theorem 1. $g_*: \pi_1(X,x_0) \to \pi_1(Y,y_0)$ is an onto homomorphism. Notation. A lasso at x_0 is a loop $\alpha^{-1}\beta\alpha$ where α is a simple arc and β is a loop lying entirely within one of the A_i. Theorem 2: Ker g_* is generated by the homotopy classes in X of the lassoes. Theorem 3: Each 1 simplex of Y determines a component $A'_{ij} \neq \emptyset$ of $A_i \cap A_j$ and A''_i of A_i respectively and a homomorphism $\pi_1(A'_{ij},a'_{ij}) \to \pi_1(A'_i,a'_i)$. The group Ker g_* is determined

by the groupoid of edges of Y and the corresponding homomorphisms. Corollary. If the A'_{ij}'s and A''_i's are simply connected, then $\pi_1(X,x_0)$ is isomorphic to $\pi_1(Y,y_0)$. When $n = 2$ and $A_1 \cap A_2$ is connected our result gives the usual form of the classical Van Kampen Theorem [Seifert-Threlfall, Lehrbuch Der Topologie, Chelsea]. (Received September 10, 1958.)

548-171. K. W. Kwun: Characterization of the n-sphere.

Let X be a compact triangulable space that is of dimension n at each point. We describe a class S(X) of upper semicontinuous decompositions of X such that for each element s of S(X) the decomposition space s(X) is homeomorphic to X. Each s is described as the limit of the stars of what we call "cellular decompositions" of X. It is shown that s is n-monotone, i.e., each element of s has the trivial homology. Here we use the Cech homology theory with a field coefficient group. In order to show that the class S(X) has the desired property, it turns out that it is enough to characterize the euclidean closed n-cell in a certain fashion. Our characterization needs some preliminary definitions, and therefore, it is impossible to give a full description here. But it is based on a certain subdivisibility and uses the notion of the pseudo-sphere, introduced by R. L. Wilder for teaching purposes, and some theorems of Bing. (Received July 7, 1958.)

THE OCTOBER MEETING IN PRINCETON, NEW JERSEY

October 25, 1958

549-1. J. G. Kemeny: A probability limit theorem requiring no moments.

Let X be any lattice-valued random variable taking on values a_i with probability p_i. Let S_n be the sum of n independent random variables which have the distribution of X. It is shown that if a, a', b, and t are chosen so that the 2 probabilities below are positive, then $\lim_{n\to\infty}(\Pr[S_{n-b} = nt + a]/\Pr[S_n = nt + a'])$ always exists, and has the value $h(t)^{a'-a}/E\{h(t)^X\}^b$. The function h(t) may be obtained as follows. From the generating function $f(s) = \sum p_i s^{a_i}$ we define $g(s) = sf'(s)/f(s)$. This is defined on some interval (restricted to positive numbers) $s_1 < s < s_2$, and is monotone increasing on this interval. If $g(s_1) < t < g(s_2)$, then we define $h(t) = g^{-1}(t)$. If $t \leq g(s_1)$, then $h(t) = s_1$; and if $t \geq g(s_2)$, then we let $h(t) = s_2$. It is shown that the results of Chung and Erdos, in Probability limit theorems assuming only the first moment I, are consequences of the above theorem.

549-2. B. L. Reinhart: Harmonic integrals on foliated manifolds.

Let V be a compact C^∞ manifold of dimension p + q, foliated by a family of p-dimensional differentiable submanifolds; then there are local coordinates $(x^1,\ldots,x^p,y^1,\ldots,y^q)$ with (x) parametrizing the submanifolds. A form is baselike if it has the local expression $\sum \phi_{\alpha\ldots\nu}(y)\, dy^\alpha \wedge \ldots \wedge dy^\nu$. Let d'' be the restriction of the exterior derivative to such forms. Assume there exists a metric locally of the form $\sum g'_{ij}(x,y)\omega^i\omega^j + \sum g''_{\alpha\beta}(y)dy^\alpha dy^\beta$, where $\{\omega^i\}$ define a subspace complementary to $\{dy^\alpha\}$. Define *'', δ'', and Δ'' operating on base-like forms by using the tensor $\sum g''_{\alpha\beta}dy^\alpha dy^\beta$. Cover V by coordinate systems such that $dx^1 \wedge \ldots \wedge dx^p \wedge (g'')^{1/2} dy^1 \wedge \ldots \wedge dy^q$ is well-defined; Δ'' is self-adjoint in the Hilbert space defined by using this measure. Theorem. The d'' cohomology of base-like forms is isomorphic to the solution space of Δ''. The proof uses the technique of Reinhart, Trans. Amer. Math. Soc. vol. 88, pp. 243-276. Corollary. These groups are finite dimensional, and the dimension of the r-th group is equal to that of the (q-r)-th. (Received August 4, 1958.)

549-3. H. S. Shapiro. Extremal measures, linear functionals, and Tchebycheff approximation.

The following theorem is proved: Let $\phi_1,\dots,\phi_n$ be linearly independent functions, real valued and continuous on a compact subset S of some Euclidean space, and let $a_1,\dots,a_n$ be real numbers not all zero. Among those real measures μ on S satisfying $\int \phi_i d\mu = a_i$, $i = 1,\dots,n$ there is one μ^* such that (i) $\int |d\mu^*|$ is a minimum and (ii) μ^* is a purely jump measure whose carrier contains at most n points. This theorem implies that, if T is a bounded linear functional on the linear space of functions $f = \sum c_i\phi_i$, normed by $\|f\| = \max|f|$, then $T = \sum_1^r \alpha_i T_i$, where $r \leq n$, the T_i are "elementary functionals" $f \longrightarrow f(P_i)$, and $\sum|\alpha_i| = \|T\|$. It also implies some known results on Tchebycheff approximation. Extensions to the complex-valued case are also given. (Received May 7, 1958.)

549-4. W. F. Stoll: About the concept of meromorphic maps.

Let G and H be complex spaces, $A \subseteq G$, and $M = G - A$ closed and thin. A holomorphic map τ: $A \longrightarrow H$ is called gapless if a subsequence of $\tau(P^\nu)$ converges for every convergent sequence $P^\nu \in A$ with $\lim_{\nu\to\infty} P^\nu \in M$. A holomorphic map τ: $A \longrightarrow H$ is called (weakly) meromorphic if $\sum(P,L) = \{Q \mid \lim_{\nu\to\infty}(P^\nu, \tau(P^\nu)) = (P,Q)$ with $P^\nu \in L \cap A\}$ consists of one (at most one) point for every $P \in M$ and every one-dimensional complex submanifold L of G when $L \cap M = \bar{L} \cap M = \{P\}$. According to R. Remmert τ is called R-meromorphic if $\overline{\{(P,\tau(P)) \mid P \in A\}}$ is analytic in $G \times H$. R-meromorphic maps are weakly meromorphic. It is unknown if both concepts of meromorphy are equivalent. A sufficient condition is given by the following theorem: If the set of singularities of τ is thin, of codimension p - 1 and contained in a countable union of sets which are thin of codimension $p \geq 2$, if H is connected and p-dimensional, then τ is R-meromorphic, if at least one of the following conditions holds: (1) τ is meromorphic and a non-constant holomorphic function exists on H; (2) τ is meromorphic and p-1 independent meromorphic functions exist on H; (3) τ is gapless and H is not compact. (Received June 24, 1958.)

549-5. Leon Greenberg: Geometric proofs of two theorems in group theory. Preliminary report.

For integers n, α, β, γ, let F(n) be the free group of rank n, $A(n) = \langle a_1,a_2,\dots,a_n,b_1,\dots,b_n; \prod_{i=1}^n a_i b_i a_i^{-1} b_i^{-1} = 1\rangle$, $B(n) = \langle a_1,a_2,\dots,a_n; \prod_{i=1}^n a_i^2$

$= 1\rangle$, and $C(\alpha,\beta,\gamma) = \langle r_1,r_2,r_3;\ r_1^2 = r_2^2 = r_3^2 = (r_1r_2)^\alpha = (r_2r_3)^\beta = (r_3r_1)^\gamma = 1\rangle$. The two theorems are: (1) Let U and V be finitely generated subgroups of one of the groups F(n), A(n), B(n), $C(\alpha,\beta,\gamma)$. In the last case, suppose U and V have no elements of finite order. Then $U \cap V$ is finitely generated. (2) Let G be one of the groups: F(n); A(n), $n > 1$; B(n), $n > 2$; $C(\alpha,\beta,\gamma)$, $1/\alpha + 1/\beta + 1/\gamma < 1$. Let H be a finitely generated subgroup which contains a subnormal subgroup of G. If $G = C(\alpha,\beta,\gamma)$, suppose H contains no elements of finite order. Then the index [G:H] is finite. The above groups are realized as groups of linear fractional transformations which leave the unit disk invariant. A theorem of J. Nielsen, concerning such groups, is used to prove the following two theorems, which imply (1) and (2). (1') Let U and V be finitely generated hyperbolic subgroups of a discrete group. Then $U \cap V$ is finitely generated. (2') Let G be a finitely generated discrete group with limit points L_G, and let H be a hyperbolic subgroup with limit points L_H. If $L_H = L_G$ and H is finitely generated, then the index [G:H] is finite. (Received June 26, 1958.)

549-6. H. W. Milnes and R. B. Potts: Stability criteria for the numerical solution of partial differential equations by boundary contraction.

The numerical solution is considered of linear homogeneous, second order partial differential equations of the form $A(r)u_{rr} + 2B(r)u_{r\theta} + C(r)u_{\theta\theta} + D(r)u_r + E(r)u_\theta + F(r)u = 0$ referred to a coordinate system in the plane topologically equivalent to the polar system. The solution surface is assumed to exist interior to the unit circle and to satisfy either Dirichlet or Cauchy data prescribed at the boundary. A mesh system is defined consisting of mesh points on a sequence of concentric circles converging to the origin. A numerical technique is introduced which determines from the given data the first circle of the unknown values in the immediate neighborhood of the boundary. The next inner circle is determined from the newly computed data and the solution then proceeds progressively inwards. Necessary and sufficient conditions for the stability of this boundary contraction procedure are given. Applications to both hyperbolic and elliptic equations are discussed. (Received July 14, 1958.)

549-7. I. S. Gál: Multiplication and division of distributions.

Let Y be the linear space of infinitely differentiable functions y on the space of reals S. Let X be the space of those $x \in Y$ which have compact supports. Let $\mathcal{U}$ be the uniform structure of Schwartz distributions or the similar weaker

structure defined by finite sets of X. Then the map A: $X \to X$ where $Ax = ax$ with $a \in Y$ is uniformly continuous; hence it can be extended to a continuous map A: $\overline{X} \to \overline{X}$ where $\overline{X}$ is the completion of X, i.e. $\overline{X}$ is the space of distributions with respect to $\mathcal{U}$. Suppose that $a \in Y$ has only isolated zeros $s_1, s_2, \ldots \in S$. Let $v_1, v_2, \ldots \in X$ have disjoint supports and such that $v_k(s) = 1$ at $s = s_k$ $(k = 1,2,\ldots)$. If each zero is of order one then there is a unique distribution $Ax \in \overline{X}$ for every $x \in \overline{X}$ such that $Ax \perp v_k$ for $k = 1,2,\ldots$ and $a(Ax) = x$. This map A is continuous. Let D: $\overline{X} \to \overline{X}$ denote differentiation and let $v \in X$ with $(v,1) \neq 0$. Then for every $x \in \overline{X}$ there is a unique $Ax \in \overline{X}$ such that $Ax \perp v$ and $DAx = x$. This map A is continuous. Similar results hold when $a \in Y$ has only isolated zeros of finite order and when D denotes an n-th order linear differential operator with analytic coefficients. Continuity and uniqueness is assured by increasing the number of orthogonality conditions. (Received July 28, 1958.)

549-8. Walter Koppelman: On the spectral theory of singular integral operators.

Consider an operator of the form $Lx(\lambda) \equiv f(\lambda)x(\lambda) + (\pi i)^{-1}\int_a^b (k(\lambda)k(\mu))^{1/2}(\mu-\lambda)^{-1}x(\mu)d\mu$ defined on $L^2(a,b)$, where $f(\lambda)$ and $k(\lambda)$ are real, continuously differentiable functions on the closed interval $[a,b]$, the functions $f'(\lambda) \pm k'(\lambda)$ have only finitely many zeros, and $k(\lambda) > 0$, $a \leqq \lambda \leqq b$. The spectrum of such operators is shown to be the interval $\min\{f(\lambda) - k(\lambda)\} \leqq \xi \leqq \max\{f(\lambda) + k(\lambda)\}$. The following criterion for the existence of a simple continuous spectrum is established: For each ξ within the interval between $f(a) - k(a)$ and $f(b) + k(b)$ each of the equations $\xi = f(\lambda) + k(\lambda)$, $\xi = f(\lambda) - k(\lambda)$ has at most one solution λ, $a < \lambda < b$; for every real ξ in the exterior of this interval, one of the equations $\xi = f(\lambda) + k(\lambda)$, $\xi = f(\lambda) - k(\lambda)$ has at most two solutions λ, $a < \lambda < b$, while the other has none. (This criterion remains valid even when $k(\lambda)$ vanishes on a set of measure zero.) A complete spectral representation for the operator L is obtained in this case. The method developed here also gives complete spectral representations for certain operators L whose spectra have higher multiplicities. (Received August 22, 1958.)

549-9. Hewitt Kenyon and A. P. Morse: Runs.

A run is such a (not necessarily transitive) relation that corresponding to each x and each y in its domain there is such a z in its domain that everything

which comes after z also comes after both x and y. Every direction is a run but runs, unlike directions, can handle topological convergence. Runs operate on a essentially different and, we feel, more convenient level than filter bases. Moreover runs have a wider scope in application than filter bases. A theory of limits is developed; applications are given. This paper will appear in the Pacific Journal of Mathematics. (Received August 25, 1958.)

549-10. Solomon Leader: On clusters in proximity spaces.

A proximity space is a set X with a binary relation "A is close to B" on subsets of X subject to certain axioms. (Efremovich: The geometry of proximity, Mat. Sb. N. S. vol. 31 (73)(1952) p. 189) A cluster c from X is a maximal class of close subsets of X which also has A or B in c whenever $A \cup B$ is in c. (1) Every pair of close sets belongs to some cluster. (2) X is compact iff every cluster has a point. (3) For X a subspace of Y every cluster c from X is part of a unique cluster d from Y consisting of all subsets of Y close to every set in c. Every cluster d from Y to which X belongs contains a unique cluster c from X consisting of all subsets of X in d. (4) X is a dense subspace of a unique compact Hausdorff space $\overline{X}$ with two sets close iff their closures intersect. (Smirnov On proximity spaces, loc. cit. p. 543) (5) Every proximity-preserving f on X has a unique extension to a continuous mapping from $\overline{X}$ onto $\overline{fX}$. The Urysohn lemma and Stone-Weierstrass theorem are thereby applied to proximity spaces. (Received August 26, 1958.)

549-11. R. D. McWilliams: Closed extensions of unbounded linear transformations.

Let Z and Y be real Banach spaces, and let L be an unbounded linear transformation from a dense subspace X of Z into Y. In an attempt to generalize Friedrichs' extensions of differential operators in Hilbert space [Comm. Pure Appl. Math. vol. 7 (1954) pp. 345-392], two extensions of L, the strong extension L_S and the adjoint extension L_A, are defined in the following way (subject to suitable conditions): suppose that $\{x_n\} \subset X$, $x_n \to z \in Z$; if $Lx_n \to y \in Y$, then $y = L_S z$; if $y^*(Lx_n) \to y^*(y)$ for all $y^* \in Y^*$ for which $y^*(Lx)$ is continuous as a function of x, then $y = L_A z$. The definition of L_A is due to Ficken (unpublished). It is shown, by generalizing an argument of Riesz and Sz.-Nagy [Functional analysis, pp. 305-306], that L_S and L_A coincide with the minimal closed extension of L (if at least one of the three extensions exists). Certain examples are

given, and the relation with Friedrichs' extensions is discussed. (Received September 2, 1958.)

549-12. Bernard Epstein: The kernel-function and conformal invariants.

Let $K_D(z,\zeta)$ denote the Bergman kernel-function of a domain D of the complex plane, and let $\lambda_D^1(t), \lambda_D^{01}(t), \lambda_D^{001}(t)$, $t \in D$, denote the minima of the integral $\iint_D |f(z)|^2 dx\,dy$ over all functions analytic and possessing uniform integrals in D under the following restrictions, respectively: (a) $f(t) = 1$ (b) $f(t) = 0, f'(t) = 1$ (c) $f(t) = 0, f'(t) = 0, f''(t) = 1$. These "minimum integrals" can be expressed in terms of the kernel-function and its derivatives (cf. S. Bergman, The kernel function, Mathematical Surveys V, pp. 24-30) and the combinations $(\lambda^1)^2/\lambda^{01}$, $(\lambda^1)^3/\lambda^{001}$ are readily shown to be conformal invariants closely related to the "invariant metric" defined by the quadratic differential $K_D(z,z)|dz|^2$. Our principal result is the following: If the two aforementioned invariants can be determined at any single point of a doubly-connected domain D, the modulus of D (ratio of radii of the conformally equivalent annulus) can be computed. This extends a result of Zarankiewicz in ZAMM, vol. 14 (1934) who showed that the modulus can be determined if the first of these invariants is known throughout D. It appears plausible that for domains of higher connectivity the moduli can also be expressed in terms of (a greater number of) invariants formed from "minimum integrals". (Received September 2, 1958.)

549-13. L. B. Robinson: Solution of a problem proposed by I. M. Sheffer.

Given the functional equation (1) $u(x,y;\lambda,\mu,\nu) = f(x,y) + \lambda\int_1^x M(x_1,y)\,u(x_1^p,y^p;\lambda,\mu,\nu) \times d(1/(1+x_1)) + \mu\int_1^y N(x,y_1)u(x^p,y_1^p;\lambda,\mu,\nu) \times d(1/(1+y_1)) + \nu\int_1^x\int_1^y K(x_1,y_1)u(x_1^p,y_1^p;\lambda,\mu,\nu) \times d(1/(1+y_1))d(1/(1+x_1))$, $|x| \leqq 1, |y| \leqq 1$, $p = 1,2,3$ and its derivative with respect to y. Seek to satisfy them by the formulae $u(x,y;\lambda,\mu,\nu) = \Delta(x,y;\lambda,\mu,\nu)/D(x,y;\lambda,\mu,\nu)\,du/dy = (d/dy)(\Delta/D)$, Δ and D are finite and integral in λ,μ,ν. Such a solution is impossible. Dr. I. M. Sheffer encouraged the author to seek a solution positive or negative. (Received September 2, 1958.)

549-14. B. Brainerd and Joachim Lambek: Rings of quotients of Boolean rings.

The maximal ring of quotients [Y. Utumi, Osaka Math. J. vol. 8 (1956) pp. 1-18; G. D. Findlay and J. Lambek, Canadian J. Math. vol. 1 (1958) pp. 77-85

etc] of a Boolean ring is its Dedekind-MacNeille cut completion [G. Birkhoff, Lattice theory, p. 58]. (Received September 4, 1958.)

549-15. Lee Lorch and Peter Szego: A singular integral whose kernel involves a Bessel function. II.

Let $t^h f(t)$ be Lebesgue-integrable over $(0,A)$ for some constant h, where $A > 1$, and suppose that $f(1-)$ and $f(1+)$ both exist. It is shown here (i) that $(C,1)\text{-}\lim \int_0^A f(t) pJ_p(pt)dt = (1/3)f(1-) + (2/3)f(1+)$ as p becomes infinite, where J_p is the Bessel function of the first kind and p-th order. The proof is carried out by showing that the Lebesgue constants, $\Lambda_1(p)$, associated with the $(C,1)$ transform of the kernel $pJ_p(pt)$ are uniformly bounded. For the Lebesgue constants, $\Lambda(p)$, corresponding to the untransformed kernel, it is proved (ii) that $\Lambda(p) = (2/\pi)^{3/2}[\int_1^A (t^2 - 1)^{-1/4} dt]\, p^{1/2} + O(1)$ as p becomes infinite. Finally, (iii) the Gibbs phenomenon for this kernel is discussed. The work reported her continues that published in the Duke Math. J. vol. 22 (1955) pp. 407-418, vol. 24 (1957) p. 683. (Received September 8, 1958.)

549-16. F. B. Jones and G. S. Young: Product spaces in n-manifolds.

The following theorem for product spaces appears not to have been noticed Let $A_1, A_2, \ldots, A_n$ be a nondegenerate locally compact connected sets. Suppose that $P = A_1 \times A_2 \times \ldots \times A_n$ can be imbedded in an n-manifold, M^n. Then each A_i is either an arc, a simple closed curve, a ray, or an open curve. Except when P is an n-torus not all of the sets, A_i, can be simple closed curves. Furthermor our theorem holds true in n-dimensional generalized manifolds. (Received September 8, 1958.)

549-17. George Whaples: Quadratic lattices over local fields.

In fields complete under a discrete valuation with residue class field of characteristic not 2, problems of equivalence of quadratic forms over the field or over its integral domain completely reduce to similar problems in the residue class field. For such fields the converse of Witt's theorem is true: if equivalence classes of forms are characterized by dimension, determinant, and Hasse invariant, then every form in five variables has a nontrivial zero. For arbitrary fields, I know no counter example to this converse of Witt's Theorem it is true for all fields in which every form in 6 variables with determinant -1 has a nontrivial zero. (Received September 8, 1958.)

549-18. P. T. Church: Boundary images of meromorphic functions.

Let f be a function meromorphic on the open unit disk D and continuous on $\overline{D}$. The image of the boundary of D is a Peano space, and G. MacLane (Proc. Amer. Math. Soc., 1955) asked whether every Peano space is so representable. In an earlier abstract Notices Amer. Math. Soc., August, 1958, p. 472, the author cited a counter-example; he now has a sufficient condition for a Peano space to have this property. The proof is primarily topological, and the following extension of a theorem of Stoïlow (Leçons sur les principes topologiques de la théorie des fonctions analytiques) is used as a lemma: If f is a function light interior on D and continuous on D, and if the image of the bounding circle is neither a single point nor the entire sphere, then $f = g \circ h$, where h is a homeomorphism of D onto itself, h continuous on $\overline{D}$, and g is a function meromorphic on D and continuous on $\overline{D}$. (Received September 9, 1958.)

549-19. R. T. Prosser: Representations of *-algebras.

By a *-algebra we mean a linear associative algebra admitting a conjugate-linear anti-automorphic involution. Gelfand and Neumark have shown that under suitable conditions a Banach *-algebra may be faithfully represented as an algebra of bounded operators acting on a Hilbert space, with the adjoint operation as involution. Here we extend their results by showing that under similar conditions an arbitrary *-algebra may be faithfully represented as an algebra of possibly unbounded operators acting on a common dense domain in a Hilbert space, with the restricted adjoint operation as involution. Criteria are given for essential self-adjointness and boundedness of the representing operators. Applications to Lie algebras, partial differential equations, probability and quantum mechanics are discussed. (Received September 9, 1958.)

549-20. Leonard Gross: Integration and nonlinear transformations in Hilbert space.

An algebra, A, of functions on a real Hilbert space, H, is described with the property that each function in A corresponds in a natural way to a measurable function on the probability space, P, associated with H in the theory of integration over H, for which theory see I. E. Segal, Tensor algebras over Hilbert spaces, Trans. Amer. Math. Soc. vol. 81 (1956) pp. 106-134. The functions in A are characterized entirely by their continuity properties with respect to a suitable topology on H. A convergence theorem for functions in A is proved which

relates convergence in a suitable sense on H to convergence in probability of the corresponding measurable functions. In case H is finite dimensional A is all continuous functions. In a third theorem sufficient conditions are given for a nonlinear transformation, T, on H to induce a new measure on P which is equivalent in the sense of absolute continuity to the original measure. In the finite dimensional case the new measure is $m \cdot T^{-1}$ if the original measure is m. This theorem is analogous to Theorem II of R. H. Cameron and W. T. Martin, The transformation of Wiener integrals by nonlinear transformations, Trans. Amer. Math. Soc. vol. 66 (1949) pp. 253-283. (Received September 9, 1958.)

549-21. E. S. Rapaport: Groups with L-polynomials.

Certain extensions, E, of groups, A, by the free cyclic group F_1 are examined. E is assumed to have an L-polynomial $L(x) = l_0 + l_1x + \ldots + l_nx^n$, $A = E' =$ commutator subgroup of E and E has a presentation on more generators than relations. This includes the knot groups. I prove: $L(1) = \pm 1$; if A is free and finitely generated, then $A = F_n$ and l_0 and l_n have absolute value 1; If A is finitely generated, then A/A' is torsion-free of rank n; if l_0 and l_n have absolute value 1 then A/A' is finitely generated; if L(x) is a polynomial for which l_0, l_n, and L(1) are units, then there is an extension E of F_n by F_1 with L-polynomial L(x) and with $F_n = E'$ and all presentations of E on n+1 generators and n relations are Nielsen-isomorphic; when $n = 2$, the group E of the last-mentioned theorem has a presentation on two generators and one relation. The chief tool is the free differential calculus. (Received September 10, 1958.)

549-22. D. E. Spencer: On the Liénard potentials.

The Liénard potentials seem to be generally accepted as giving the correct retarded ϕ and A caused by a moving point charge. It is customarily believed that they satisfy the wave equations and lead to the Maxwell equations. This paper shows, however, that two of the four Maxwell equations are not satisfied exactly by the Liénard potentials. Discrepancies in the derivation of the Liénard potentials are pointed out and discussed. (Received September 10, 1958.)

549-23. H. F. Trotter: On the "product" of semi-groups of operators.

Let T_t, T'_t be two semi-groups of operators on a Banach space, of class (C_0), and with infinitesimal generators A, A' respectively. (For terminology, see E. Hille and R. S. Phillips, Functional analysis and semi-groups, Amer.

Math. Soc. Colloquium Publications, vol. 31, rev. ed., 1957.) Define $S_{a,t} = \lim_{h \to 0} (T_h T'_{ah})^n$ where a is positive and n is the greatest integer in th^{-1}. If T_t and T'_s commute for all s,t, the closure of A + aA' generates the semi-group $S_{a,t} = T_t T'_{at}$. If $\|T_t\|, \|T'_t\| \leq 1$ and $D(A) \subset D(A')$, the operators $S_{a,t}$ are everywhere defined and form a semi-group generated by A + aA', for all positive a in some neighbourhood of 0. There exist two groups of class (C_0) with $\|T_t\|, \|T'_t\| = 1$ and $D(A) = D(A')$, such that the closure of A + A' does not generate a semi-group of class (A). This answers a question raised on page 417 of the above reference. (Received September 10, 1958.)

549-24. S. B. Kochen: Filtration systems. I.

Let $\{A_i | i \in I\}$ be a sequence of similar relational systems. A subset F of the direct product $\prod_{i \in I} A_i$ is called a filtration system over the A_i if (i) F is closed under the operations of $\prod_{i \in I} A_i$; and (ii) for every prime dual ideal D in the Boolean algebra B(F) formed by the class of all sets $\{i | \phi(f_1(i),\ldots,f_n(i))$ is valid in $A_i\}$, where $\phi(x_1,\ldots,x_n)$ ranges over the elementary formulas of the A_i and the f_j's range over the elements of F, $\phi(f_1/D,\ldots,f_n/D)$ is valid in F/D if and only if $\{i | \phi(f_1(i),\ldots,f_n(i))$ is valid in $A_i\} \in D$. Here, F/D is the relational system whose domain consists of equivalence classes f/D of elements f of F, where the equivalence is given by $f \equiv g$ if and only if $\{i | f(i) = g(i)\} \in D$; the fundamental relations of F/D are given by $\langle f_1/D,\ldots,f_n/D\rangle \in R$ if and only if $\{i | \langle f_1(i),\ldots,f_n(i)\rangle \in R_i\} \in D$, where R_i is the corresponding relation in A_i. If all $A_i = A$ and F is a filtration system, then F/D is an arithmetically equivalent extension of A. $\prod_{i \in I} A_i$ forms a filtration system, with $B(\prod_{i \in I} A_i) = 2^I$. This gives an algebraic construction of nonstandard models of the integers. A number of results from the theory of models previously proved by means of the Gödel Completeness Theorem have been proved by D. Scott and the writer using the fact that $\prod_{i \in I} A_i$ is a filtration system. (Received September 10, 1958.)

549-25. Richard Block: On Lie algebras of rank one.

Let F be an algebraically closed field of characteristic >3. Every known simple Lie algebra over F of rank one is either 3-dimensional or an Albert-Zassenhaus algebra, i.e., an algebra with a basis $\{u_\alpha | \alpha \in G\}$, where G is a finite additive subgroup of F, and multiplication $u_\alpha u_\beta = \{\alpha h(\beta) - \beta h(\alpha) + \alpha - \beta\} \cdot u_{\alpha+\beta}$, where h is any additive functional on G [Albert and Frank, Rend. Sem. Mat. Torino, vol. 14, p. 138]. THEOREM. If L is a Lie algebra of dimension

>3 over F, with a 1-dimensional Cartan subalgebra such that for every nonzero root α, multiplication between L_α and $L_{-\alpha}$ is nondegenerate, then L is an Albert-Zassenhaus algebra. The proof uses the work of Kaplansky on rank one algebras [to appear in Trans. Amer. Math. Soc.] to reduce the problem to the case in which the roots form a group and the root spaces are one-dimensional. The results of Chang on the representations of the Witt algebra [Abh. Math. Sem. Univ., Hamburg, vol. 14] are also used. The converse of the theorem is trivially true. (Received September 11, 1958.)

549-26. Albert Edrei and W. H. J. Fuchs: On the growth of meromorphic functions.

Let $f(z)$ be a meromorphic function of lower order μ ($< \infty$) and let p be the integer defined by $p - 1/2 \leqq \mu < p + 1/2$. Let $N(r,a)$ and $T(r,f)$ be the usual symbols of Nevanlinna's theory and assume $\lim\sup_{r\to\infty}\{N(r,0) + N(r,\infty)\}/T(r,f) \leqq \epsilon/A(p+1)$, where $0 < \epsilon \leqq 1$ and A is an absolute constant. Then $1 \leqq p$ and the order of $f(z)$ is less than $p + 1/2$ so that, if $E(u)$ denotes the primary factor of genus p, the function $f(z)$ is representable in the form (I) $f(z) = z^k \exp(\alpha_0 z^p + \alpha_1 z^{p-1} + \ldots + \alpha_p) \prod E(z/a_\nu)/\prod E(z/b_\nu)$. Putting $c(r) = \alpha_0 + p^{-1}\{\sum_{|a_\nu|\leqq r} a_\nu^{-p} - \sum_{|b_\nu|\leqq r} b_\nu^{-p}\}$, the inequality $|T(r,f) - |c(r)|r^p/\pi| < \epsilon|c(r)|$ is valid for all sufficiently large values of r. Let $f^*(z)$ be the function obtained by performing in (I) the following changes: (i) the arguments of the quantities a_ν and b_ν are modified arbitrarily but their moduli are left unchanged; (ii) the α's are modified arbitrarily. Let $c^*(r)$ denote the quantity analogous to $c(r)$, formed with the corresponding elements of $f^*(z)$. Then $|T(r,f^*) - |c^*(r)|r^p/\pi| < \epsilon|c(r)|$ provided r is sufficiently large. (Received September 11, 1958.)

549-27. G. D. Findlay and Joachim Lambek: Multiplicative ideal theory of a noncommutative ring.

An ideal A of a ring R (with unity) is called (right)-integral if $rA = 0$ implies $r = 0$ for all r in R. It is called right invertible in an extension S of R if there exists an ideal A^{-1} in S such that $R \subseteq A^{-1}A$ and $AA^{-1} \subseteq R$. If A is right invertible in any extension of R it is right invertible in the maximal ring of right quotients of R defined by Y. Utumi [Osaka Math. J. vol. 8 (1956) pp. 1-18] and the present authors [Canadian J. Math. vol. 1 (1958) pp. 77-85, 151-167]. Let R be integrally closed in the following sense: if q is an element of the maximal

ring of right quotients of R such that $qA \subseteq A$, A integral, then $q \in R$. An integral ideal C is called classical if it is the product of right invertible maximal ideals. If A is an integral ideal such that the integral ideals containing A satisfy the maximum condition, then A is uniquely expressible in the form BC where C is classical and B is not contained in any proper classical ideal. (Received September 11, 1958.)

549-28. R. P. Gosselin: On Diophantine approximation and trigonometric interpolating polynomials.

Let $I_{n,u}(x;f)$ be the trigonometric polynomial of order n which takes on the values of a given function f at the points $u + 2\pi k/(2n + 1)$, $k = 0, 1, \ldots, 2n$. Theorem: Let Ψ be any monotone increasing function defined on $(0,\infty)$. There exists a function f such that $\Psi(|f|)$ is integrable and such that for almost every point (x,u), the sequence $I_{n,u}(x;f)$ diverges. The theorem generalizes a result of Marcinkiewicz and Zygmund (Fund. Math. vol. 28 (1937)), and the proof is effected by results on Diophantine approximation related to a theorem of Kronecker. (Received September 11, 1958.)

549-29. E. V. Haynsworth: Generalizations of Brauer theorems on stochastic matrices.

Brauer [Limits for the characteristic roots of a matrix IV, Duke Math. J. vol. 19 (1952) pp. 75-91] proved a number of theorems about the characteristic roots of stochastic matrices, several of which are generalized in this paper, for matrices which can be partitioned into stochastic blocks, and other types of partitioned matrices. Special transformations are introduced which simplify the problem of finding eigenvalues for certain types of matrices which arise frequently in problems in physics and chemistry. (Received September 11, 1958.)

549-30. Hilary Putnam and Martin Davis: Uniqueness ordinals and extended R-systems.

A system $\mathcal{C}^*$ of notations for ordinals in higher constructive number classes, adopted from Kreider's system $\mathcal{C}$, is studied. The transfinite hierarchy H_a is extended to allow $a \in \mathcal{C}^*$, and it is shown that each ordinal with a notation in $\mathcal{C}^*$ is a uniqueness ordinal, in the obvious sense. This result both includes and extends Spector's uniqueness theorem for the system S_3 and also gives a uniqueness result for S_1. The notion of extended R-system is defined

and it is shown that $\mathcal{L}^*$ is a maximal extended R-system. (Received September 11, 1958.)

549-31. J. L. Selfridge and E. G. Straus: On the determination of numbers by their sums of a fixed order.

Let $\{x\} = \{x_1,\ldots,x_n\}$ be a set of complex numbers and let $\{\sigma\} = \{\sigma_1,\ldots,\sigma_t\}$ be the set of sums of s distinct elements of $\{x\}$. This paper considers to what extent $\{x\}$ is determined by $\{\sigma\}$ and what sets can be $\{\sigma\}$ sets. The first n elementary symmetric functions of $\{\sigma\}$ can be prescribed arbitrarily and they determine $\{x\}$ uniquely in these cases: (a) $s = 2$, $n \neq 2^k$ (b) $s = 3, n > 3$, $n \neq 6, 27, 486$ (c) $s = 4$, $n > 4$, $n \neq 8, 12$ (d) $n \nmid s!s^n$. Two different sets $\{x\}$ and $\{y\}$ can have the same $\{\sigma\}$ in these cases: (a) $s = 2$, $n = 2^k$ (b) $n = 2s$. If $\{\sigma\}$ does not determine $\{x\}$ uniquely then n must satisfy a certain diophantine equation $f(n,k) = 0$ and the first k elementary symmetric functions of $\{\sigma\}$ must satisfy an algebraic equation. Whether $\{\sigma\}$ always determines $\{x\}$ if $s = 3$, $n = 27$; $s = 3$, $n = 486$ or $s = 4$, $n = 12$ remains open. The principal method is to obtain $\sum x_i^k$ recursively for $k = 1,\ldots,n$ from $\sum \sigma_i^k$. (Received September 11, 1958.)

549-32. I. J. Schoenberg: Some polynomial extremal problems and mechanical quadratures.

Let $\alpha(x)$ be a nondecreasing function with infinitely many points of increase in the finite or infinite range (a,b) such that the mass-distribution $\int_a^b d\alpha(x)$ has moments of all orders. Given the natural integer n, denote by π_n the class of polynomials P(x) of degree not exceeding n and subject to the following conditions: $P(x) \geq 0$ if $a < x < b$, $\int_a^b P(x)d\alpha(x) = 1$. The problem is to determine the function $M(z) = \sup P(z)$ for $P \in \pi_n$, $-\infty < z < \infty$. For the special case when $n = 2k$ is even, $(a,b) = (-1, +1)$, $\alpha(x) = x$, and $z = +1$, $M(z)$ was determined by S. Bernstein (Leçons sur les propriétés extrémales...,1926, pp. 48-50). See also A. Papoulis, Optimum filters with monotonic response, Proc. of the Institute of Radio Engineers vol. 46 (1958) pp. 606-609. The general problem is solved here by means of ideas and results on formulae of mechanical quadrature which should be associated with the names of Christoffel, Radau, Fejér and Shohat. (Received September 11, 1958.)

549-33. Lamberto Cesari: Quasi additive set functions and the concept of integral.

An axiomatization of the concept of integral over a continuous variety is discussed which comprehends recent more particular developments. A collection $\{I\}$ of subsets I of a space A and a family $\mathcal{D}$ of finite systems $D = [I] = [I_1,\dots,I_N]$ are taken into consideration together with a "mesh" $\delta(D)$, $D \in \mathcal{D}$. A mesh $\delta(D)$ is any function defined for each $D \in \mathcal{D}$ such that (m1) $0 < \delta(D) < +\infty$; (m2) given $\epsilon > 0$ there are $D \in \mathcal{D}$ with $\delta(D) < \epsilon$. Thus $\delta(D)$ is an index of fineness of D, and is not required to decrease by refinement. Real vector functions $\phi(I) = [\phi_1,\dots,\phi_k]$ are considered defined for each $I \in \{I\}$, and the concepts of quasi additivity, quasi subadditivity, and total variation $V(\|\phi\|)$ are introduced. Mappings T: $p = T(w)$, $w \in A$, $p = (x_1,\dots,x_m) \in K \subset E_m$ are considered. If $\omega(I) = \operatorname{osc}(T,I)$, $I \in \{I\}$, $\omega(D) = \max \omega(I)$ for all $I \in D$, then the continuity requirement $\omega(D) \leqq \delta(D)$ is made. Functions $f(p,q)$, $p \in K$, $q = (q_1,\dots,q_k)$, are considered satisfying the usual conditions of continuity in p,q and positive homogeneity in q. Then, if $V(\|\phi\|) < +\infty$, the limit exists and is finite $I(f,T,\phi) = \lim \sum_{I \in D} f[p(\tau),\phi(I)]$, $\tau \in I$, as $\delta(D) \to 0$ (integral of f with respect to T and ϕ). Under further axioms (not involving absolute continuity) $I(f,T,\phi)$ is proved to be given by an integral expression involving Radon-Nikodym derivatives and a measure function ψ associated to $\|\phi\|$. (Received September 15, 1958.)

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Performance of the work requires close association with scientists of other laboratories, operations research personnel of all branches of the Armed Services, and with senior military and civilian personnel.

Studies undertaken by this group will provide guide lines for the hardware research of future years. Staff members are expected to initiate ideas in support of a broad program of National Defense needs and carry them through appropriate analyses with assurance that sound results will be given consideration by the responsible agencies.

The Laboratory's locale, equidistant between Baltimore and Washington, D. C., allows staff members to select urban, suburban or rural living and either of these two outstanding centers of culture as a focal point for fine living.

These appointments offer exceptional opportunities. For information and arrangements for interview, write in confidence to:

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